

THE
BRITISH YOUTH'S
INSTRUCTOR:

Or, A NEW and EASY GUIDE to
PRACTICAL ARITHMETIC.

WHEREIN THE

Rudiments of common ARITHMETIC, VULGAR
and DECIMAL FRACTIONS, the Extraction and
Use of the SQUARE and CUBE ROOTS, &c.

Are so easily Treated of, and so plainly Demonstrated,
That any Person may, of himself, (in a short Time) become
acquainted with every Thing necessary to the

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Shewing how to measure any regular Piece of Timber, Tiling,
Thatching, Brick-work, or Piece of Land; as also how to guage
any Cistern, Piece of Malt, or common Cooler, Tub, &c.

And to this Edition is now first added,

A compendious METHOD of BOOK-KEEPING,
Whereby any one capable of subtracting one Sum of Money from ano-
ther, may state and balance their own Accounts correctly, and have
a clear View of all their Transactions.

The Whole designed for such as have hitherto neglected, or have not
had an Opportunity of becoming acquainted with Figures; and at-
tempted in natural and familiar Dialogues, in order to render the
Work more easy and diverting, as well as useful to Learners.

RECOMMENDED BY SEVERAL

Eminent WRITING-MASTERS and ACCOMPTANTS.

By DANIEL FENNING,

Author of the Young ALGEBRAIST'S COMPANION; Description and
Use of the GLOBES (both in Dialogues); the Universal SPELLING-
BOOK; Royal English DICTIONARY; Young Man's BOOK of
KNOWLEDGE; a new English GRAMMAR; and a new Treatise on
MENSURATION.

The TENTH EDITION, revised, corrected, and improved,
By W. BURBIDGE, SCHOOLMASTER, in *Salisbury*.

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M.DCC.LXXXIII.

[Price bound Three Shillings.]

WILLIAM B. ELLIOTT
M.D.
1873

DEDICATION

To the SCHOOLMASTER

GEORGE BROWN and
JAMES BROWN

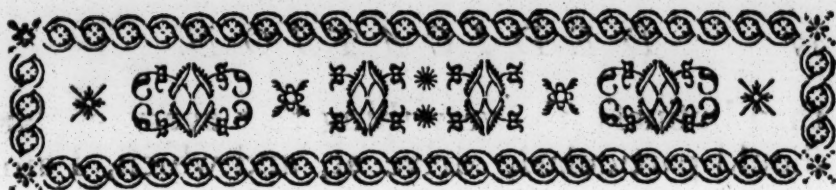
NEW YORK

THE
SCHOOLMASTER
ENCOURAGED THE AUTHOR

TO REMAIN YOUR DEAR FRIEND
AND UNDEVELOPED

I am very sensible (and I think you are too)
many other hands have written in this book
before me, and have to thank you for
the which cannot be expressed in words
but as you are kind and I am
I think that this book





DEDICATION

To the SCHOOLMASTERS of

Great-Britain and Ireland.

GENTLEMEN,

THE great demand for, and speedy
T sale of, the five last impressions of
this small Treatise, naturally leads
me to think, that many of you have
encouraged the undertaking.

Permit me, therefore, in this sixth edition,
to return you hearty thanks for your friendly
and undeserved favours.

I am very sensible (as I told you before) that
many abler hands have undertaken this task
before me, and have, in large volumes, done
that which cannot be expected to be found
here: but as you are sensible too many of them
have spent their time rather upon curiosities
A 2 than

than business (the very thing that every scholar should particularly drive at, and which, I am sensible, every one of you would promote the knowledge of) I have here made it my chief care to inform the learner of every thing that is necessary thereto.

It was not without reluctance that I appeared upon this subject; but having published a small Treatise of FRACTIONS and ALGEBRA, by way of dialogue, in 1759,* which has met with great encouragement, I was persuaded by several Schoolmasters, and private gentlemen, to publish also a Piece of ARITHMETIC, after the same manner; as they were sensible, they said, (and indeed, I confess, I think the same) that this way of writing conveys the matter better, and communicates things sooner to the learner, than the bare setting of sums, and not working them at all, or in a dry intricate manner.

As I do not pretend to recommend the work by comparing it with other authors, so, I hope, you will not condemn it, till you have perused it throughout.

'Tis true, that the last method of reducing *C's. qrs. lbs.* into *lbs.* (in the 1st example of *Tare and Tret*) is well worthy your observations, it being as easy as it is new; and several persons in business, to whom I have communicated it,

* The fourth Edition, with some Quadratic Problems, was published in 1772.

D E D I C A T I O N.

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never pretend to use any other way for ease and expedition.

To such of you, indeed, as are perfect masters of the sciences, this little piece may appear insignificant; but to others, I am sensible, the Dialogues will be of great service, as they are intermixed with a variety of friendly examples, in natural conversation; and I persuade myself, they will be of great help to your pupils, and, in a great measure, ease you of that heavy task, which every diligent master (with a number of scholars) must of necessity labour under.

This, Gentlemen, is one reason (as I said) why I built it upon this plan; and I hope, for the design's sake, you will forgive those errors that have yet escaped my notice, and which you know are so common to a work of this sort; though I have taken care to correct all such as I have at present discovered in the former editions.

I have made no great alteration in the work, only have taken off part of the Double Rule of Three *Direct* and *Reverse*, (as you know the Rule of Five Numbers will supply that defect) in order to make more room (by the desire of several merchants) for a fuller and more clear explanation, both of the true and customary way of Discount.

To my thanks, I add my best wishes for your success, jointly and separately, in your several undertakings; and desire always to subscribe myself,

GENTLEMEN,

Your most obliged humble Servant,

D. FENNING.

*Royal Exchange Assurance-Office,
London, Sept. 1, 1767.*

PRE-



P R E F A C E

TO THE

F I F T H E D I T I O N.

KIND READER,

I Here present you with the fifth edition of my small Treatise of ARITHMETIC, which I have improved, and endeavoured to render as plain as time, room, and opportunity would admit of.

For such of you who have too much neglected this branch of education, and for others who have neither time nor opportunity to apply to a proper master, the following work was chiefly designed, and was at first carried on no farther than the *Rule of Three Direct*: But I considered with myself, that it might fall into the hands of many, who would be glad to have a notion of *Fractions*. I have, therefore, to serve you, treated upon most of the Rules of *Arithmetic*, and for the sake of those that live in the country in particular, I have given some instruc-

instructions to measure timber, or a common piece of ground; as also to guage a cask or piece of malt, and several other useful and practical examples, which, I am persuaded, will be of service to you.

One thing, which is seldom taken notice of in a PREFACE, I would have every one of you, that are learners, to observe, and that is, that if, upon the trial of any sum or question, you do not find it the same as the answer, do not let that discourage you; for if you examine your own work after you have done it, and are so much master of the rules as to know when you are right or wrong, you may conclude you are right, whatever answer you may find in my work, or any other author's: but before you determine this, mind and be sure your own work is right.

As to such of you that have little or no notion of figures, if you have a mind to learn, I am sensible you may very easily do it, with little or no trouble; for I have taken the more pains, that you might have the less: and if it does but answer the end of instructing such of you as are quite ignorant, and improving others that have already some knowledge of figures, I shall look back upon the undertaking with pleasure, notwithstanding those mean criticisms and remarks that may be made upon such oversight, which can hardly be avoided in a work of this sort.

But

But every impartial reader will consider the nature and design of the work; for 'tis the plan upon which it is built that is to be minded: if *this* be plain and easy (as I hope it is) there is no fear but the learner will find a sensible satisfaction, and the work will be crowned with success.—As for such *Carpers* that are resolved to amuse themselves with the bone only, they are extremely welcome; but let them be civil, and not snarl at those who would eat the flesh quietly.

I am,

Kind READER,

Your humble Servant,

And Well-wisher,

D. FENNING.

London, Sept. 1, 1767.

But



P R E F A C E

TO THIS

E D I T I O N.

THE number of editions this work has been honoured with must be an evident proof of its *utility*. Mr. *Fenning* the original author, has now paid the debt which all must pay. But, where is that man who can say no one can improve upon what I have submitted to the eye of the public. The question seems to answer itself. Having therefore received so much encouragement from the editions already published, the Proprietors were stimulated to make it more universally useful.

Persons who have learnt to work the several rules of Arithmetic may be desirous of knowing the *Art of Book-Keeping*; and it has been suggested, that something of that kind would be a considerable addition. In compliance with this intimation, amidst a variety of other improve-

Improvements, we have added, *A concise Method of keeping Books*, which Mr. BURBIDGE, the present Editor, has taught with the greatest success for several years, and found always approved of by those who have received his pupils into their 'Compting-Houses. Mr. BURBIDGE, in this respect, has not only anticipated the wishes of his peculiar friends, but has gone beyond them, as he has made many corrections in the body of this work, which, we hope, will meet with the patronage of those who wish to encourage an enlargement, or amendment, of any work, which has been honoured with so many repeated proofs of its utility.

June 1, 1783.

To the PUBLIC.

WE, whose Names are hereunto subscribed, having perused the plan of this little Treatise of ARITHMETIC, do allow the dialogue to be very well adapted to the purpose; and as the rules and examples are laid down in a natural, easy, and familiar manner, we beg leave to recommend it, as the most useful and easy book for learners extant.

Edmund Anguish, <i>Accomptant</i>	Abraham De Lire, <i>Philom.</i>
James Barclay, <i>Writing-master</i>	Thomas Newbery, <i>ditto</i>
William Bently, <i>Surveyor</i>	John Quant, <i>Writing-master</i>
George Coles, <i>ditto</i>	Wm. Richardson, <i>ditto</i>
Mr. Coulthist, <i>of the Academy,</i>	David Rowland, <i>Accomptant</i>
<i>Prescot-street, Goodman's-fields</i>	John Smyth, <i>Writing-master</i>
Henry Deacon, <i>Accomptant</i>	Jonas Smith, <i>ditto</i>
Randal Evans, <i>Writing-master</i>	Thomas Smithe, <i>ditto</i>
Anthony Gilbert, <i>Surveyor</i>	John Smythe, <i>Accomptant</i>
Edward Griffiths, <i>ditto</i>	Zachary Snaper, <i>ditto</i>
Samuel Hill, <i>Philom.</i>	Erasmus Turner, <i>ditto</i>
T. Humphreys, <i>Writing-master</i>	J. Thompson, <i>Writing-master</i>
Thomas Hughes, <i>ditto</i>	James Thurston, <i>ditto</i>
Samuel Hornby, <i>ditto</i>	Daniel Trunker, <i>ditto</i>
Timothy Langley, <i>Accomptant</i>	William Thorley, <i>ditto</i>
Abraham Longden, <i>ditto</i>	James Thorpe, <i>ditto</i>

To the RECOMMENDERS.

GENTLEMEN,

I Return you hearty thanks for the honour you have done me, by the favour of your names to this little Treatise: let me crave your further assistance in noting down those errors that you may occasionally find, and you will still further oblige,

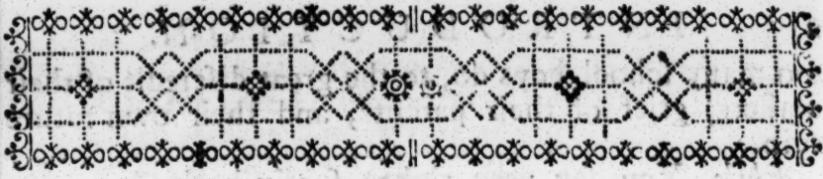
GENTLEMEN,

Your very humble Servant,

Sept. 24, 1767.

DANIEL FENNING





AN EASY GUIDE
TO
Practical ARITHMETIC.

The INTRODUCTION.

CHAP. I.

DIALOGUE I.

Between PHILO, a Tutor, or Master, and TYRO, a young Scholar ; concerning the Rudiments of Arithmetic.

TYRO visits PHILO.

Tyro.



DEAR Sir, I am your very humble servant—You will pardon me, I hope—I hear you have done instructing *Tyrunculus*, and I am come to lay claim to a former promise of your giving me a better notion of common *Arithmetic*.

Philo. You please me very much, young *Tyro*, I assure you, to see you so willing to learn ; but I hear you understand the first four rules already.

Tyro. When I was at school I had some knowledge of them, as I thought ; but it was not well grounded ; and when I left school, instead of practising at home, and making myself master of what I learnt there, I bent my mind to play and idleness, like other naughty boys, and were it not for your kind offer, I should know the want of it too late. I choose, therefore, sir, to begin at the very lowest branch, that I may see the reason of what I am doing, and not learn by rote, as

B

too

INTRODUCTION.

too many school-boys do, to the great discredit of their masters, grief of their parents, and their own future ruin.

Philo. You say very right; for in beginning again you will be confirmed in what you know already; for I shall proceed with you the same as if you never had began, that others may be the better informed; therefore, for their sakes do not you be angry, if I should dwell upon some things longer than you may think there is occasion for, since I tell you the reason before hand.

Tyro. Far be it from me, sir, to take it amiss; for though I do know something of the first four Rules, I am sensible there are many thousands who know nothing of the matter, and you do well to consider them also.

Philo. I am glad, *Tyro*, to see you so considerate; it gives me great hopes of your being master of what I am about to instruct you in.



DIA-

DIALOGUE II.

SECTION I.

Of NUMERATION, ADDITION of *Whole Numbers*,
Money, Weights, and Measures, &c.

Tyro. **W**HAT is *Numeration*, and what does it teach?

Philo. *Numeration* is the true distinction and pronunciation of number; that is, it teaches us to write down, read, and express any number or numbers whatsoever. For the better understanding of which, observe the following table.

N. B. The letter *C* stands for an hundred, and *X* for ten.

TABLE.

Places	C of Millions.		X of Millions.		C of Thousands.		X of Thousands.		Hundreds.		Tens.		Units.	
	9	8	7	6	5	4	3	2	1					
	9	8	7	6	5	4	3	2	1					
		9	8	7	6	5	4	3	2					
			9	8	7	6	5	4	3					
				9	8	7	6	5	4					
					9	8	7	6	5					
						9	8	7	6					
							9	8	7					
								9	8					
									9					

NOTE I.

This table you ought to get by heart, at least, so as to understand the nature of it.

Cast your eye now, *Tyro*; upon it; you see that there are nine places of figures from units to hundreds of millions. All the figures under the first

row are units; those in the second row towards the left-hand, are under the place of tens; all in the third row are called hundreds; those in the fourth row thousands, &c.

Now, in order to know the value of, or how to express any number in the table, I begin at the top figure towards the right-hand, and say units (1);

4 *Of NUMERATION, ADDITION, &c.*

then units, tens, (twenty-one, 21;) then units, tens, hundreds, (three hundred and twenty-one, 321;) thus I go on, and find the fourth figure (viz. 4) under the place or name of thousands, and accordingly, I call it four thousand, which joined to the other three figures, will be four thousand, three hundred, and twenty-one, 4,321. The fifth figure being in the place of tens of thousands, is thus read: fifty-four thousand, three hundred, and twenty-one, 54,321. Thus proceed, till you come to the last figure of all towards the left-hand (which stands in the place of hundreds of millions) and you will easily perceive, that those nine figures are thus expressed: Nine hundred eighty-seven millions, six hundred fifty-four thousand, three hundred and twenty-one, 987,654,321. Read this once more, and observe the commas or stops that are put to the figures, for they answer to, or correspond with the stops in the words that are written out at length.

NOTE 2.

You are further to observe, *Tyro*, that the *Numeration-Table* is not always set with these figures just in the form they here stand; for had they been any other nine figures, they are numbered and expressed after the same manner: For instance, suppose they were 1,2,3,4,5,6,7,8,9, this is expressed after the same manner, only instead of 987 million, it is now 123 million; instead of 654 thousand, it is here 456 thousand; and instead of 321, it is now 789. So also 999,999,999, is read thus: Nine hundred ninety-nine million, nine hundred ninety-nine thousand, nine hundred and ninety-nine, &c. So that you see it is only having a due regard to the places under which the figures stand; for you may see that after the first three places of figures (viz. units, tens, and hundreds) the next three places have the name of thousands, the next three the name of millions, as you may observe by the following Table, which, I believe, will be of service.

TABLE

TABLE 2.

Millions Place.			Thousands Place.			Hundreds.	Tens.	Units.
1	2	3.	4	5	6.	7	8	9
	1	2.	3	4	5.	6	7	8
		1.	2	3	4.	5	6	7
			1	2	3.	4	5	6
				1	2.	3	4	5*
					1.	2	3	4
						1	2	3
							1	2
								1

Now here you see they are divided by stops or periods, which is certainly a help to the expressing the numbers: thus, suppose I would express the number against which this mark (*) is placed, I find the *twelve* stands under the place of thousands, so that I say it is twelve thousand, three hundred and forty-five, &c.

Tyro. I perceive it, sir, very plainly; but pray what use are cyphers of; or are they of no signification?

Philo. Cyphers are of no signification when they stand alone, (thus 000, or 0000, all stand for nothing :) nor are they of any signification placed before any figure or figures; thus, 02 is but 2; and 0005 is but 5 still: but when cyphers are put after figures, it makes the number ten, twenty, thirty, a hundred, or a thousand times more in value. Thus 1, by adding a cypher, it is (10) ten; add two cyphers, thus, 100, it is an hundred, &c. as by the following table.

4 *Of NUMERATION, ADDITION, &c.*

then units, tens, (twenty-one, 21;) then units, tens, hundreds, (three hundred and twenty-one, 321;) thus I go on, and find the fourth figure (*viz.* 4) under the place or name of thousands, and accordingly, I call it four thousand, which joined to the other three figures, will be four thousand, three hundred, and twenty-one, 4,321. The fifth figure being in the place of tens of thousands, is thus read: fifty-four thousand, three hundred, and twenty-one, 54,321. Thus proceed, till you come to the last figure of all towards the left-hand (which stands in the place of hundreds of millions) and you will easily perceive, that those nine figures are thus expressed: Nine hundred eighty-seven millions, six hundred fifty-four thousand, three hundred and twenty-one, 987,654,321. Read this once more, and observe the commas or stops that are put to the figures, for they answer to, or correspond with the stops in the words that are written out at length.

NOTE 2.

You are further to observe, *Tyro*, that the *Numeration-Table* is not always set with these figures just in the form they here stand; for had they been any other nine figures, they are numbered and expressed after the same manner: For instance, suppose they were 1,2,3,4,5,6,7,8,9, this is expressed after the same manner, only instead of 987 million, it is now 123 million; instead of 654 thousand, it is here 456 thousand; and instead of 321, it is now 789. So also 999,999,999, is read thus: Nine hundred ninety-nine million, nine hundred ninety-nine thousand, nine hundred and ninety-nine, &c. So that you see it is only having a due regard to the places under which the figures stand; for you may see that after the first three places of figures (*viz.* units, tens, and hundreds) the next three places have the name of thousands, the next three the name of millions, as you may observe by the following Table, which, I believe, will be of service.

TABLE

T A B L E 2.

Millions Place.

1 2 3.
1 2.
1.

Thousands Place.

4 5 6.
3 4 5.
2 3 4.
1 2 3.
1 2.
1.

Hundreds.
Tens.
Units.
7 8 9
6 7 8
5 6 7
4 5 6
3 4 5*
2 3 4
1 2 3
1 2
1

Now here you see they are divided by stops or periods, which is certainly a help to the expressing the numbers: thus, suppose I would express the number against which this mark (*) is placed, I find the *twelve* stands under the place of thousands, so that I say it is twelve thousand; three hundred and forty-five, &c.

Tyro. I perceive it, sir, very plainly; but pray what use are cyphers of; or are they of no signification?

Philo. Cyphers are of no signification when they stand alone, (thus 000, or 0000, all stand for nothing :) nor are they of any signification placed before any figure or figures; thus, 02 is but 2; and 0005 is but 5 still: but when cyphers are put after figures, it makes the number ten, twenty, thirty, a hundred, or a thousand times more in value. Thus 1, by adding a cypher, it is (10) ten; add two cyphers, thus, 100, it is an hundred, &c. as by the following table.

TABLE 3.

1	One.
10	Ten.
100	An hundred.
1000	A thousand.
10000	Ten thousand.
100000	An hundred thousand.
1000000	A million.
10000000	Ten million.
100000000	An hundred million.
1000000000	A thousand million.
10000000000	Ten thousand million.
100000000000	An hundred thousand million.
1000000000000	Million of millions.

And thus you may go on as far as you please.

Tyro. I perceive it; but who could tell the value, or express so many figures out of the course of the table?

Philo. You talk like a learner indeed. You see the twelfth figure, by the above table, has the name of an hundred thousand millions; therefore were there 3, 4, or 5 times, or 5 hundred times as many figures, it would be as easy to number them in order, as well as if there were but 5 in all.

Tyro. That is a little surprising to me, I confess; for some think they do great things to number 9 or 10 figures only.

Philo. Well, *Tyro*, we will not call that pride altogether; for pride is good in learners so far as it tends to emulation only; that is, an earnest desire to excel in learning: but pray observe, suppose I had ever so many figures to number; you plainly see, by the last table, that the seventh figure is millions place, and that the thirteenth place has the name of millions of millions; so also the nineteenth figure would be millions of millions of millions: But as the word millions would be repeated so often in a very large number of figures, as to render it tiresome, there is a shorter and much easier way of expressing the number by certain words, which answer to every seventh figure,

figure, and to any degree of millions, as appears by the following Table.

T A B L E 4.

Dots.

1	The 7th figure from the units place is millions.
2	13 is bimillions, or millions of millions.
3	19 is trimillions, or the 3d deg. of millions.
4	25 is quartrillions, or the 4th deg. of millions.
5	31 is quintillions, or the 5th deg. of millions.
6	37 is sexquillions, or the 6th deg. of millions.
7	43 is septillions, or the 7th deg. of millions.
8	49 is octillions, or the 8th deg. of millions.
9	55 is nonmillions, or the 9th deg. of millions, or millions nine times repeated, &c.

Suppose it were required to number the 57 following Figures, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321.

Here I make a period or dot over every seventh figure, and find there are nine dots in all: Then searching in the above table, I find the ninth dot to bear the name of nonmillions, the eighth of octillions, &c. Therefore, I number the above figures thus; 321 nonmillions, 987 thousand; 654 octillions, 321 thousand; 987 septillions, 654 thousand; 321 sexquillions, 987 thousand; 654 quintillions, 321 thousand; 987 quartrillions, 654 thousand; 321 trimillions, 987 thousand; 654 bimillions, 321 thousand; 987 millions, 654 thousand, 321. And thus you may go on to as many more places, &c. Or you may number them by the second part of the table, and that is, by putting small figures where the dots stand, as 1, 2, 3, 4, till you come to 9: Then begin, and say 321⁹ of the 9th degree of millions, 987 thousand; 654 of the 8th degree of millions, &c. till you come to the end: And thus, by putting small figures over every seventh figure, you may number to the 20th, 30th, or any higher degree of millions whatsoever.

8 *Of NUMERATION, ADDITION, &c.*

Tyro. I understand you well ; and though it is likely there may never be so many figures in any sum ; yet, it is good to know how to number them when a question is asked, though it be for fancy's sake only. But now, *Philo*, if you please, I will ask you a few questions, which I am at a loss to know.

Quest. 1. How is *eleven thousand, eleven hundred, and eleven*, set down in 5 figures ?

Philo. I know this is a common question, and it is easily done, if you consider ; for this is the answer, 12,111, viz. twelve thousand, one hundred and eleven.

Proof by ADDITION.

Eleven thousand is	11,000	} add
Eleven hundred is	1,100	
Eleven is	11	
	<hr/>	

Ans. 12,111, as above.

Quest. 2. How is *fifteen thousand, fifteen hundred, and fifteen*, set down in five figures ? *Answer* 16,515

But you are to observe, *Tyro*, that though these oddities are easily answered, you see ; yet I would not have you be concerned with them ; because they are very badly expressed, and are an abuse of language : for, suppose you stood indebted to me, and should ask me what it was, and I should say fifteen pounds, fifteen shillings, and fifteen pence, would you not think it much better to say fifteen pounds, sixteen shillings, and three pence ? Take this then for a rule, *Tyro*, that, that is the best method of expression, that is the shortest, freest, and most natural. Therefore, though the aforesaid numbers run a little smoother in words than what I have now mentioned, (the reason of which is, because they are each but one whole number) yet is the sense of expressing fifteen thousand, fifteen hundred, and fifteen, very little better than fifteen pounds, fifteen shillings, and fifteen pence.

☞ Now there are several things a little dark in expression, relating to numbers, which are actually of use

use when they are known. I imagine, *Tyro*, you now and then read good books: You remember we read in the book of Kings, and in Isaiah, that an Angel destroyed in the camp of the Assyrians, an hundred, and fourscore and five thousand. How do you set this down properly?

Tyro. Why really, sir, I am at a loss; and I have seen older persons than myself puzzled at it.

Philo. So have I; but it must be for want of considering. Is not four score and five the same as 85? Therefore, the number is one hundred eighty-five thousand, thus, 185,000.

Again, you read in the Revelations, of a multitude of ten thousand times ten thousand; which is 100,000,000, viz. one hundred million: and a little farther you read of another, consisting of two hundred thousand thousand, which is, 200,000,000, viz. two hundred million, which you will see demonstrated, Example the 3d and 4th, in compendiums of multiplication.

I shall finish Numeration with some of the old Roman numbers, as it may be of service.

A TABLE of Old ROMAN Numbers.

L. signifies or stands for (50) fifty.

C. for (100) hundred.

CC. (200) two hundred.

CCC. (300) three hundred.

CCCC. (400) four hundred.

D. signifies (500) five hundred,

or thus IƆ.

DC. stands for (600) six hundred,

or thus IƆC.

DCC. (700) seven hundred,

or thus IƆCC.

DCCC. (800) eight hundred,

or thus IƆCCC.

DCCCC (900) nine hundred,

or thus IƆCCCC.

M. (1000) a thousand,

or thus CƆ.

MM. (2000) two thousand, &c.

ROMAN Numbers explained by FIGURES.

DL. 550.

DCCX. 710.

MDXIV. 1514.

MDCLI. 1651.

MDCCIX. 1709.

MDCCLXXXIII. 1783, &c.

And now, *Tyro*, we will proceed to addition.

S E C T I O N II.

Of ADDITION.

Tyro. WHAT do you mean by Addition?

Philo. Addition signifies the gathering, collecting, and adding together, two or more numbers into one sum.

Tyro. How many parts are there in Addition?

Philo. Two, simple and compound.

Tyro. What is simple Addition?

Philo. Simple Addition is that which consists of one simple or single name; that is to say, of whole numbers only, as pounds sterling, tons, yards, ells, or ounces, &c. For ten is ten, and a thousand is a thousand, in any number and quantity; though the quality or name be different; and they are all added by one rule, namely, by casting out the tens, and setting down in every line what is over ten, as you will see by and by.

Tyro. What does compound addition consist of?

Philo. Of money, weights, and measures, viz. Avoirdupois Weight, Troy Weight, Apothecaries, and Goldsmith's Weight, &c. Likewise, Dry Measure, Liquid Measure, Cloth Measure, Square Measure, Long Measure, Land Measure, and Time. Of these in their order.

S E C T I O N III.

ADDITION of WHOLE NUMBERS.

THE rule to be got by heart is,
For every ten in the units place, or first row of figures, you must carry one to the tens place, or second row, and so proceed; because ten units make ten, ten tens an hundred, and ten hundred a thousand.

Add 2	3	2	9
1	2	1	7
1	1	4	8
5	2	3	and 4 together.
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
9 <i>Ans.</i>	8 <i>Ans.</i>	10 <i>Ans.</i>	28 <i>Ans.</i>

Here

Here I say 5 and 1 is 6, and 1 is 7, and 2 is 9; which I place under the numbers, and it is done: but to prove whether it be right, I begin at top, and cast it downwards, and find it comes to the same.

Secondly, If it amounts to just ten and no more, (as in Example 3) set it down, as before; only set the cypher under the row, and set the figure of 1 out towards the left hand. So also, in Example 4, I find it amounts to 28, which I set under the row, viz. the units 8 under the row itself, and the two towards the left hand in the place of tens.

Thirdly, When you have two or three rows of figures, then, according to the rule, add up the first row, or units place, and observe how many tens it contains, and if it comes just to even tens, set a cypher underneath, and carry as many ones to the next row, as there were tens in it; that is, if it be 20, carry 2, if it be 30, carry 3, if 40 carry 4, and for 50 carry 5, &c. And the last row of all set down what it amounts to, as if it were one single row only.

An EXAMPLE at large.

Add 87625 pounds

94915

62194

76547

81965

92198

and 71964 together

567408 *Ans.*

Now observe once for all. I begin and say, 4 and 8 is 12, and 5 is 17, and 7 is 24, and 4 is 28, and 5 is 33, and 5 is 38; this is the amount of the first row, or units place, but I must not set down the whole 38, but see how many tens it contains, and find it to be 3 tens and 8 over; this 8 I set under the row, and carry 3 to the next row, or tens place, saying, 3 that I carry and 6 is 9, and 9 is 18, and 6 is 24, and 4 is 28, and 9 is 37, and 1 is 38, and 2 is 40: this being just 4 tens, I set down a cypher, and carry 4 for the 4 tens to the next row, or hundreds place, saying 4 and 9 is

B 6

13, &c.

Of ADDITION.

13, &c. and I find the third row to be 44, which is 4 tens and 4 over; therefore I set down 4, and carry 4 to the fourth row, or place of thousands, and find that it amounts to 27, then I set down the odd 7, and carry 2 to the 5th or last row, and find that it amounts to 56, and because this is the last row, I set down the whole 56, that is, the 6 under the row, and the 5 to the left-hand.

Fourthly, When there are several numbers to be added together, consisting of some fewer and some of more figures, they are added after the same manner as before, only observe, in setting down the numbers on your slate, or in a book, that you be careful to set units under units, tens under tens, or else you will be puzzled in casting them up.

Ex. 6. Suppose I were to add 3417, 26, 184, 9, 271, and 3 together: I set them down as follows, which will be a standing rule for any thing of the like nature.

EXAMPLE 6.

Loads.

3417
26
184
9
271
3
3910

Here you see I set units under units, and tens under tens, and then I cast them up from row to row as before directed.

Tyro. I understand it very well; but I should have set it down thus, with cyphers, to supply the vacant places:

3417
0026
0184
0009
0271
0003
3910

Philo.

Of ADDITION.

13

Philo. It is quite superfluous; for you have been told already, that cyphers before figures do not at all increase the value; besides they are neither so lightly, nor advantageous to cast up, nor are they seldom or ever used by accomptants, even in addition of money, to fill up any line or space whatsoever, as you will see in example 13, of *Addition of Money*.

More Examples in whole Numbers.

<i>Busbels.</i>	<i>Ells.</i>	<i>Yards.</i>
471756	6715	65432
434176	46	2145
621985	2	234
942176	2176	67
354219	200	8
471625	6	76
982196	10	354
853294	1756	3256
<hr/>	<hr/>	<hr/>
5131427	1	76258
	10912	147830

And now, *Tyro*, I will set you two or three questions by way of exercise.

Questions to exercise Addition of Whole Numbers.

Quest. 1. From *London* to *Rumford* is 11 miles, from *Rumford* to *Brentwood* 6, from *Brentwood* to *Chelmsford* 11, from *Chelmsford* to *Witham* 8, from *Witham* to *Kelvedon* 4, from *Kelvedon* to *Colchester* 10, from *Colchester* to *Manningtree* 9, and from *Manningtree* to *Harwich* 12; how many miles then is it from *London* to *Harwich*? *Answer* 71.

Tyro. I apprehend the question; it is only setting down the numbers as they stand in their order, as under.

	<i>Miles.</i>
From <i>London</i> to <i>Rumford</i>	11
From <i>Rumford</i> to <i>Brentwood</i>	6
From <i>Brentwood</i> to <i>Chelmsford</i>	11
From <i>Chelmsford</i> to <i>Witham</i>	8
From <i>Witham</i> to <i>Kelvedon</i>	4
From <i>Kelvedon</i> to <i>Colchester</i>	10
From <i>Colchester</i> to <i>Manningtree</i>	9
From <i>Manningtree</i> to <i>Harwich</i>	12
<hr/>	<hr/>
From <i>London</i> to <i>Harwich</i>	71 <i>Ans.</i>

Quest. 2 A farmer has seven fields containing the following acres, viz.

	Acres.	
In one field are	25	} How many acres are there in all? <i>Ans.</i> 192.
In another	57	
In another	18	
In another	25	
In another	15	
In another	43	
In another	9	

Ans. 192 acres.

Quest. 3. A draper has 6 pieces of cloth as under.

	Yards.	
*N ^o 1 containing	87	} How many yards are there in all? <i>Ans.</i> 210.
2 ———	45	
3 ———	17	
4 ———	8	
5 ———	32	
6 ———	21	

Ans. 210 yards.

Quest. 4. How many days are there in the year, or in the 12 calendar months? First set them down as follows:

	Days.	
1 st , January has	— 31	} <i>Note, Every fourth year is called leap year, and has 366 days; February having then 29 days.</i>
February —	— 28	
March —	— 31	
April —	— 30	
May —	— 31	
June —	— 30	
July —	— 31	
August —	— 31	
September —	— 30	
October —	— 31	
November —	— 30	
December —	— 31	

Ans. 365 days.

* *Note,* That in marking any sort of goods, N^o stands for Number. *Quest.*

Of ADDITION.

15

Quest. 5. How far is it from *London* to *Carlisle* in *Cumberland*, when

From <i>London</i> to <i>Newcastle</i> , is	149
From <i>Newcastle</i> to <i>Preston</i>	62
From <i>Preston</i> to <i>Lancaster</i>	21
From <i>Lancaster</i> to <i>Penrith</i>	50
From <i>Penrith</i> to <i>Carlisle</i>	19

Ans. 301

Tyro. I understand what you have shewn me very well.

Philo. Then I shall only leave one question more for you to try at your leisure.

Add 47, 697, 5, 91707, 100000, 26300175, 500, 62, and 987654321 together? *Ans.* 1014147514.

To prove ADDITION.

Tyro. How do you prove Addition?

Philo. Two ways. *First*, When you have cast up any sum, as before directed, then begin at the top and cast downwards instead of upwards; and if the figures come the same, no doubt but the work is right; besides in things which require care, you should make a practice of casting every line, first upwards and then downwards.

The *second* way is very well for learners, but too tedious for business; but as it is customary in schools to teach it, I shall shew you the method, which is as follows.

Let us take *Quest. 5*, which amounts to 301 miles; and to prove whether this be right, I cut off the top line of figures by a stroke of the pen, thus—

$$\begin{array}{r} 149 \\ \hline 62 \\ 21 \\ 50 \\ 19 \\ \hline \end{array}$$

301 *Ans.*

152 Add this to the top 149.

301 *Proof.* See Addition of Money.

Then

Then I begin to cast the sum up again, as I did at first, (except the figures that are cut off, or that stand above the line, and find it amount to 12, which is 2, and I carry 1; then I proceed to the second row, and find it makes 15, so that this amounts in all to 152, which I place under the 301. *Lastly*, I add this middle line 152 to the top line 149, and find that they make 301, which proves the first work to be right. I shall give you a further reason, when you come to examples of *money*, why this way is not so fit for practice in business as casting the sum upward and downward. And now we will proceed to

SECTION IV.

ADDITION of MONEY.

Tyro. **W**HAT is necessary to the learning of *Addition of Money?*

Philo. These three things; *First*, the rule; then the characters; and, *thirdly*, pence-tables; all which should be perfectly got by heart before you pretend to cast up money.

1. The RULE.

For every 4 farthings carry 1 penny to the pence; for every 12 pence carry 1 shilling to the shillings, and for every 20 in the shillings carry 1 to the pounds, which are cast up by tens, as in whole numbers.

2. Of the common CHARACTERS.

Note 1. £ stands for pounds, S stands for shillings, D stands for pence. Or thus, £. s. d. pounds, shillings, pence.

Note 2. A farthing is one-fourth part of a penny, and is thus set down $\frac{1}{4}$. An halfpenny is one-half, and is thus set down $\frac{1}{2}$. Three-farthings being three-fourths, is thus set down $\frac{3}{4}$.

Note 3. 1 quarter, 1 half, and 3 quarters, are also set thus, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

3. Of

3. Of PENCE-TABLES.

TABLE 1.			TABLE 2.		
Pence		<i>s. d.</i>	Pence		<i>s.</i>
20		1 - 8	12		1
30		2 - 6	24		2
40		3 - 4	36		3
50		4 - 2	48		4
60	} is {	5 - 0	60	} is {	5
70		5 - 10	72		6
80		6 - 8	84		7
90		7 - 6	96		8
100		8 - 4	108		9
110		9 - 2	120		10
120		10 - 0			

By the help of these two tables, the first increasing by tens, or every 10 pence; and the other by even shillings, or every 12 pence, you will soon cast up any small sums, by a due regard to the following examples.

Tyro. But I hope, Sir, you will give me an example in length, by explaining it in words; for that will be of more service to me than an hundred examples without explanation.

EXAMPLE 1.			EXAMPLE 2.		
£.	s.	d.	£.	s.	d.
Add 9	- 4	- 2	Add 41	- 3	- 8
8	- 5	- 3	62	- 9	- 7
and 7	- 9	- 6 together.	and 47	- 11	- 6 together.
<hr/>			<hr/>		
24	- 18	- 11	151	- 4	- 9

EXAMPLE 3.		
£.	s.	d.
Add 95	- 16	- 2
68	- 14	- 10
and 49	- 12	- 11 together.
<hr/>		

*

* I leave example 3 undone for your practice, and will give you another table for the more easily casting up the shillings.

Philo.

Philo. You seem to be timorous, and doubt your own abilities without occasion. Come pray try at the first sum.

Tyro. I see plainly how you do *that*, because the pence all added together do not exceed 12, nor do the shillings exceed 20; therefore I set the amount of them under the row to which they belong, and find the total to be twenty-four pounds, eighteen shillings and eleven pence.

Philo. The same is to be observed in example 3. Thus I say, 6 and 7 is 13, and 8 is 21 pence. Now by the first table, 20 pence is 1s. 8d. therefore, 21 pence must be 1s. 9d. Or, by the 2d table, I ask how many times 12 I can have in 21d. and find it 1, and 9 over; therefore, I set the odd 9 down under the place of pence, and carry 1 to the shillings, saying 1 that carry and 11 is 12, and 9 is 21, and 3 is 24 shillings. Now, as 20s. make a pound, 24s. is 1l. 4s. therefore I set the 4 under the shillings, and carry 1 to the first row of the pounds, casting them up as in *Addition of whole Numbers*, and find it 15l. so is the total 15l. 4s. 9d.

TABLE 3.

Shillings.		Pounds.	s.
20	} is {	1	-
30		1	- 10
40		2	-
50		2	- 10
60		3	-
70		3	- 10
80	}	4	-
90		4	- 10
100		5	-

Please to add up the following sums.

Ex

£

45

94

76

51

25

75

368

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Ex

EXAMPLE 4.

£.	s.	d.
45	- 9	- 9
94	- 5	- 8
76	- 7	- 10
51	- 8	- 9
25	- 7	- 6
75	- 5	- 11
<hr/>		
368	- 5	- 5

EXAMPLE 5.

£.	s.	d.
249	- 3	- 10
176	- 5	- 8
649	- 8	- 6
148	- 7	- 5
219	- 5	- 7
154	- 7	- 9

EXAMPLE 6.

£.	s.	d.
652	- 5	- 8
652	- 5	- 8
652	- 5	- 8
652	- 5	- 8
652	- 5	- 8
652	- 5	- 8

Tyro. I begin at the row of pence, and find that it amounts to 53. Now, according to my *first pence-table*, 50 pence is 4s. 2d. therefore 53 pence is 4s. 5d. Or, by *table 2*, I find how many 12 pences I can have in 53 pence, and find that 4 shillings is 48 pence, therefore 53 pence must be 4s. 5d. which odd 5 pence I set down under the row of pence, and carry the 4 shillings to the row of shillings, and find it amounts to 45. Now, by *table 3*, I find 45s. to be 2l. 5s. which odd 5s. I put under the shillings, and carry 2l. to the pounds, which I cast up as in *whole numbers*, (by tens) and find the *total*, or whole amount, to be 368l. 5s. 5d.

Philo. Very well; but now you have done it, pray let me hear you say what it comes to; for I have known many school-boys not able to read what they write or cast up.

Tyro. 'Tis very true, sir, but I think it is three hundred and sixty-eight pounds, five shillings and five pence.

Philo. Very right, and I make no doubt, by your careful proceeding, but you will understand the first four rules of *Arithmetic* in a short time. I leave example 5 and 6 undone, for you to try at your leisure, after the same way and manner; and now I will try you with an example or two with farthings.

OF ADDITION.

EXAMPLE 7.			EXAMPLE 8.			EXAMPLE 9.				
£.	s.	d.	£.	s.	d.	£.	s.	d.		
47	-	8	-	4	$\frac{1}{2}$	278	-	4	-	3
15	-	5	-	10		164	-	8	-	9
17	-	4	-	8	$\frac{1}{2}$	215	-	1	-	10
49	-	9	-	6	$\frac{1}{4}$	464	-	5	-	4
15	-	10	-	7		176	-	7	-	8
14	-	7	-	9	$\frac{3}{4}$	291	-	3	-	5
97	-	3	-	5	$\frac{1}{4}$	456	-	7	-	6
14	-	9	-	6		278	-	5	-	7

Ans. 270 - 19 - 9 $\frac{3}{4}$

Tyro. I can do these as well as the other. *First*, I begin with the farthings, saying, 3 and 3 is 6, and 1 is 7, and 2 is 9, and 2 is 11 farthings, that is 2d. and 3 farthings over, which 3 farthings I set under the row of farthings (thus $\frac{3}{4}$) and carry 2d. to the pence, and find they amount to 57 pence, which is 4s. 9d. this odd 9 I set under the pence, and carry 4 to the shillings, and find them 59, that is 2 pounds, and 19 over, which 19 I set under the shillings, and carry 2 to the pounds, I find them come to 270: so is the total or answer 270l. 19s. 9d. $\frac{3}{4}$.

Philo. Very well done, indeed: proceed in the same manner, and you will find *Ex. 8*, to be 2752l. 3s. 3d. $\frac{1}{4}$.

Tyro. I am obliged to you, sir, and now I should be glad you would give me instructions how you manage double figures in the shillings.

Philo. I will shew you three or four different ways, and you may take that which appears most natural and easy.

4. *Of casting up double figures in the shillings, such as 16, 17, 18, or the like.*

EXAMPLE 10.			EXAMPLE 11.			EXAMPLE 12.		
£.	s.	d.	£.	s.	d.	£.	s.	d.
64	-	17 - 10 $\frac{1}{4}$	756	-	17 - 8	4715	-	11 - 10 $\frac{1}{2}$
47	-	14 - 11	175	-	16 - 11 $\frac{1}{4}$	1762	-	14 - 5
67	-	18 - 10 $\frac{3}{4}$	476	-	11 - 9 $\frac{1}{2}$	6471	-	14 - 6 $\frac{1}{2}$
55	-	17 - 8 $\frac{1}{2}$	187	-	17 - 10	1754	-	15 - 11 $\frac{1}{2}$
49	-	19 - 11	356	-	19 - 5 $\frac{3}{4}$	6474	-	19 - 8
64	-	15 - 5	175	-	14 - 11 $\frac{1}{4}$	1762	-	17 - 6
<hr/>			<hr/>			<hr/>		
351	-	4 - 8 $\frac{1}{2}$						

351 - 4 - 8 $\frac{1}{2}$

ME-

Of ADDITION.

21

METHOD 1. I cast the farthings up (first upwards and then downwards) and find them 6, I therefore set down $\frac{1}{2}$, and carry 1 to the pence, which are 56, which is 4s. 8d. that is 8, and I carry 4 to the shillings; but as there are two rows of shillings, or *double figures*, I first of all cast up that row, or those figures that stand towards the right hand, saying thus, 4 that I carry and 4 is 8, and 9 is 18, and 7 is 25, and 8 is 33, and 4 is 37, and 7 is 44, the first row; then I come back upon the second row, from the top to the bottom, calling every figure of 1, ten; saying, 44 and 10 is 54, and 10 is 64, (and so on) 74, 84, 94, and 10 is 104 shillings, which is 5l. and 4 shillings over, that is 4, and I carry 1 to the pounds, which I find amount to 35l. So that the total is 35l. 4s. 8d. $\frac{1}{2}$.

METHOD 2. Some like *this* way best; they cast up the first row of shillings as before, which comes to 44, that is 2l. 4s. and set the 4 shillings under the said row, and carry the 2l. to the next row of shillings, counting 1l. for every two figures in the second row, (because two tens make 20s. or 1l.) thus in the above example, there are six figures of 1 in the second row, and counting two of them for 1l. then six will be 3l. which with the 2l. that belonged to the 44 in the first row, make 5l. to be carried to the pounds, as before.

METHOD 3. Others make use of *dots*: that is, they make a dot at every 4 in the farthings, every 12 in the pence, every 20 in the shillings, and every 10 in the pounds, which method is very easy, and suitable to heads of a dull comprehension, and a bad memory; because they are not able to carry a large number of pence and shillings in their mind, but frequently carry false from figure to figure.

Note. There are different opinions concerning pointing or dotting: Mr. *Dilworth* highly recommends it; but Mr. *Fisher* calls it slovenly and unnecessary. It is not my province to determine between those gentlemen: But this I will say, that every master should try all methods, and let the scholar use *that* which is most natural and easy to his capacity. He must be very dull who can't tell that 2 tens make 20, or that 12 twenties make 240: and to stop or point at so small numbers,

numbers, as 3, 4, 5, 6, &c. ought (if possible) to be avoided; because to add up well, and learn to cast out quick, is a great step towards *Multiplication*, and it would look very odd to see a person, (after having done a long sum in that rule) dot every 10 in casting up the product. I think therefore, there is no occasion to point or dot at every 4, 10, 11, or at even twenties, when a little practice will make it easy: but indeed when you come to do by 13, 16, 19, 28, or the like, there dotting must unavoidably be used, as you will see in the first example of *Avoirdupoise Weight*, wrought at large.

N. B. *If you are obliged to use dots, mind and make them very small; if you can see them yourself, that is sufficient: but the first method, in my opinion, is best, except it be a very long sum, such as the side of a large book, a bill, or a parish rate, or duplicate, and then the following method is the most certain and expeditious, and is very easy.*

METHOD 4. *Of making and casting up long bills, PARISH RATES, &c.*

Tyro. How is the general way of casting up very long bills, rates, &c.

Philo. The rule is this: *for every 60 in the pence carry 5 to the shillings, because 60 pence make 5 shillings; and for every 60 in the shillings carry 3 to the pounds, because 60 shillings make 3 pounds; and lastly, cast the pounds up by tens, as before directed.*

Tyro. Please to give me an example at large.

Philo. I will shew you first how to make a *parish rate*, which may be of service, and if you can cast it up well, you may also cast up any long bill, by the same rule.

The Form of a PARISH RATE, &c.

Allen Anthony, <i>Esq.</i>	—	—	3	7	8	$\frac{1}{4}$
Andrews John	—	—	1	14	7	$\frac{1}{2}$
Baker James, <i>Esq.</i>	—	—	15	16	8	
Carter Thomas	—	—		17.	11	$\frac{1}{2}$
Darby Abraham	—	—	5	17	5	
Fernel Abraham	—	—		19	8	$\frac{3}{4}$
Honner Joseph	—	—	7		9.	
Kirton John, <i>Esq.</i>	—	—	21	14.	7	$\frac{1}{4}$
Longman Thomas	—	—	5		11	$\frac{1}{2}$
Lumley Edward	—	—	3	17	6	
Manning Thomas	—	—		15	10	$\frac{1}{4}$
Martin William, <i>Esq.</i>	—	—	11	11	8	$\frac{1}{2}$
Martin Job	—	—	7	17.	8	
Nicholas Abraham	—	—	2	13	9	
Norton Daniel, <i>Esq.</i>	—	—	17	10	4.	$\frac{1}{2}$
Parker Esther	—	—		8	11	
Powell Judith	—	—		5	10	
Randall Nicholas	—	—	1	17	6	$\frac{1}{2}$
Robinson Abraham, <i>Esq.</i>	—	—	15	11.	11	$\frac{1}{2}$
Robinson John	—	—	7	10		
Robinson James	—	—	1		10	
Ruggles Nathaniel	—	—	7	11	4	
Rumley Thomas	—	—		15		$\frac{1}{2}$
Solly Thomas, <i>Esq.</i>	—	—	12	11		
Sorby William	—	—		14.		
Stratly David	—	—	8		11.	
Swallow Thomas	—	—	1	17	4	
Thompson John, <i>Esq.</i>	—	—		19	11	$\frac{3}{4}$
Twist William, <i>Esq.</i>	—	—	7	7		
Walker John	—	—	1	15.	6	
Wayland Edward, <i>Esq.</i>	—	—	8	14	9	$\frac{1}{2}$
Wingate Samuel	—	—		15	7	
Worley Abraham	—	—	5	4	10	
		£.	189	19	7	$\frac{3}{4}$

First

First, I begin with the row of farthings (as in the former questions) and find them amount to 27, which is 6 pence, and 3 farthings over: which 3 I set under the farthings thus ($\frac{3}{4}$) and carry 6 to the pence.

Secondly, for the PENCE.

I say 6 that I carry and 10 is 16, and 7 is 23, and is 32, and 6 is 38, and 11 is 49, and 4 is 53, and 11 is 64: This being 4 above 60, I make a small dot close to the 11, and carry the 4 that is over to the next figure saying 4 that I carry and 4 is 8, and 10 is 18, and 11 is 29, and 6 is 35, and 10 is 45, and 11 is 56, and 4 is just 60; therefore, as there is nothing over to carry to the next figure, I say 9 and 8 is 17, and 8 is 25, and 10 is 35, and 6 is 41, and 11 is 52, and 7 is 59, and 10 is 68; that is 8 above 60, therefore I make a dot, and carry 8 to the next figure, saying 8 and 8 is 16, and 10 is 21, and 11 is 32, and 8 is 40, and 7 is 47, and 10 is 55: now 50 pence is 4s. 2d. therefore 55 pence is 4s. 7d. which odd 7 pence I set under the pence, and carry the 4 shillings to the dots, counting (as I said before) 5 shillings for every dot, which are three in number that is 15 shillings; and the 4 I carried to them is 4 shillings; this 19 I carry to the shillings, and contrast to the other methods, I now work cross ways, taking the *double figures* as I go along, *viz.* the right-hand figure first, and then the left-hand one belonging to it, counting it for 10, as follows:

Thirdly, for the SHILLINGS.

I say 19 that I carry and 4 is 23, and 5 is 28, and 1 on the left hand of it, which is always counted for 10, is 38, and 4 is 42, and 10 on the left hand belonging to it is 52, and 5 is 57, and the 10 belonging to it is 67; here according to the same rule, I make a dot and carry the odd 7 to the next figure, as I did in the pence; saying, 7 that I carry and 7 is 14, and 9 is 23, and 10 is 33, and 7 is 40, and 10 is 50, and 4 is 54, and 10 is 64; that is dot and I carry 4, to 11 is 68, and 5 is 20, and 10 adjoining it is 30, and 11 is 41, and 10 is 51, and 11 is 62, which is dot and I carry 2

the next figure, proceeding in the same manner as before directed till I come to the top of all, and find there are 39 shillings, besides the last dot, that is 1£. 19s. therefore I set the 19 under the shillings, and carry the odd 1£. to the dots (counting 3£. for every dot, because 60 shillings is 3£.) and find them 6, which is 18£. and 11 carried to them is 19£. which I carry to the first row of the pounds, casting them up by tens, as in *whole numbers*, and find them 189£. So that the total of the rate or bill is 189£. 19s. 7d. $\frac{3}{4}$. Pray, *Tyro*, run it over once more, and the method also, and you will not lose your labour.

Tyro. Sir, I see the nature of it very well; but why do you leave those vacancies in the shillings and pence, for most people fill them up with cyphers.

Philo. I know it is the common custom of schools, but it is a very idle one, nor is it so sightly or convenient; for the cyphers hinder sight, and prevent expedition in casting up. See example 6, in *whole numbers*.

5. Of CYPHERS, where necessary, and where not.

EXAMPLE 13, without cyphers, according to the true Order of BOOK-KEEPING.

EXAMPLE 13, with cyphers, according to the common Custom of SCHOOLS.

£.	s.	d.
5478	- 9	- 6
1	- 19	- 5
	- 6	- $8\frac{3}{4}$
179	-	-
	- 17	- $\frac{1}{2}$
7	- 5	- 9
1	- 17	- 7
	- 4	- $\frac{1}{2}$
<hr/>		
5670	-	$\frac{3}{4}$

£.	s.	d.
5478	- 09	- 06
0001	- 19	- 05
0000	- 06	- $08\frac{3}{4}$
0179	- 00	- 00
0000	- 17	- $00\frac{1}{2}$
0007	- 05	- 09
0001	- 17	- 07
0000	- 04	- $00\frac{1}{2}$
<hr/>		
5670	- 00	- $00\frac{3}{4}$

Now, *Tyro*, I would ask you which is most sightly, that which stands clear, or the other confused with cyphers. And not only this, but every line of the sum, and the total also, reads much better: for it is only five thousand, six hundred, and seventy pounds,

and three farthings; whereas the other (according to the manner of cyphers, and the too common custom of school-boys) must be thus read, *five thousand, six hundred, and seventy pounds, no shillings, and no pence, three farthings*. And tho' custom some years ago prevailed upon one of the most ingenious authors *, to use the *expression* after this sort; yet it does not at all justify the correctors of the last edition, since it is now quite superfluous, and out of date.

Note, Though I say it be not customary to use cyphers in *addition of money*, yet in *weights and measures* they are often made use of, and the learner may take his own fancy that teaches himself; but it is my opinion that masters in general would find it much easier and better, both to themselves and scholars, to set their addition sums quite clear, and use no cyphers at all before figures.

Tyro. I am obliged to you, sir; and now, if you please, I would know how you prove *addition*?

Philo. That is quite easy, *Tyro*.

6. To prove ADDITION of MONEY,

There are two ways, one by cutting off the top line (of which see under the proof of *addition*) but it is not practicable in business; the best way is this: begin at the bottom, and cast up to the top, noting down what it comes to; then begin at the top, and cast the same row downwards, and if it amounts to the same as before, there is no doubt but the work is right, provided you observe carefully to set the total down right

SECTION V.

Containing some farther explanation of things necessary to be known, with the manner of drawing out bills, writing notes and receipts, &c. being very proper to exercise the young beginner.

Tyro. I shall be at a loss under this section.

Philo. You are mistaken; for I shall not leave you to yourself wholly, but will explain every thing I am capable of, to your understanding; because I am

sensible

sensible many school boys can add up sums that are set them very well; but when they are called upon out of school, to write out and cast up a small bill, or the like, how awkward do they go about it, and will sit puzzling over it so long, that one would think they knew nothing at all of the matter: and when it is done, it is often set in such confusion, that it will puzzle much better scholars than themselves to cast it up; because they do not observe to place their figures right under one another, which remember, *Tyro*, you be always careful of, for it is a great advantage, and the work will look neat, and you will have the praise.

Tyro. 'Tis very true, sir, I have known several that could cast up a bill if others would figure it down, but have not had any notion of drawing it out themselves: nor can I say I have, and therefore should be obliged to you to shew me.

Philo. That I will; but first of all it will be proper to shew you some *signs* and *contractions* made use of both in arithmetic and business.

1. *Of contracted Signs and Characters used in common Arithmetic.*

Note 1. This character (+) signifies addition, and when placed betwixt numbers shews that they are all to be added together. Thus, $5 + 7 + 11 + 9$, is thus read, 5 more 7, more 11, more 9, &c.

2. This character (—) signifies less, and is the sign of subtraction, and shews the number after it is to be taken out of the number before it; thus, $11 - 7$ is 11 less 7, which is 4; and $148 - 93$ shews that 93 is to be subtracted from 148, &c.

3. This (\times) is the sign of multiplication, and signifies into, and shews that all the numbers betwixt which it is placed are to be multiplied continually into each other. Thus, $4 \times 5 \times 3$, shews that 4 is to be multiplied into or by 5, and that product by 3.

4. This ($:$) is the sign of division, and shews the number before it is to be divided by the number after

it. Thus $48 \div 6$ shews that 48 is to be divided by 6.

5. This (:) signifies to.

6. This ($::$) signifies so is.

7. The two last characters put together thus ($:: ::$) are the sign of the rule of 3 or proportion. Suppose you see $2 : 4 :: 8 : 16$; it is thus read, as 2 to 4, so is 8 to 16, &c. As $a : b :: c : d$. As a to b , so is c to d , &c.

8. This ($=$) is the sign of equality, and signifies that the figures or quantities placed before it are equal to those after it; thus $4 + 12 = 16$, is read 4 more 12 equal to 16. So also $25 - 16 = 9$, is 25 less 16, or 16 subtracted from 25 equal to 9. So also $5 \times 9 + 12 - 2 \div 5 = 11$, is thus read; 5 into 9, more 12, less 2, divided by 5, is equal to 11.

9. This ($\sqrt{}$) is the sign of the square root.

10. This ($\sqrt[3]{}$) the sign of the cube root.

2. Of contracted Words, and what they signify.

Note 1. *Bt.* stands for, or signifies *bought*.

2. *Dr.* is *debtor*. *Cr.* is *creditor*.

3. *Do.* stands for *ditto*, and signifies the same thing, place, or sort of goods, as was wrote in the line before; it is used by merchants, tradesmen, and accomptants both in books and bills, to avoid writing the same thing over and over again.

4. *Co.* signifies *company*; that is, when two or more are in trade and in partnership.

5. *Messrs.* signifies *Messieurs*, or gentlemen concerned together in some trade or business, and is used with the foregoing contraction (*Co.*) on the head or front of the bills, instead of writing all their names at length; thus suppose *John Sharp* to have a bill upon *Aaron Nelson*, *John Long*, and *Joseph Truman*, it would be ridiculous to write thus:

Mr. Aaron Nelson, Mr. John Long, and Mr. Joseph Truman, bought of John Sharp.—But we write thus:

Messrs. Aaron Nelson and Co. bt. of John Sharp, you will see in the following example.

A Line

A Linen-Draper's Bill.

EXAMPLE 1.

Messrs. *Aaron Nelson* and Co. bt. of *John Sharp*.

1783.

Jan. 4.	To 11 pieces of Irish, at 3 <i>l</i> .	}	£.	s.	d.
	per piece ———				
			33		
12.	To 2 ditto, at do. ———		6		
Feb. 8.	To long lawn as by agreement		3	15	6
Apr. 12.	To 2 pieces of check, as by do.		1	7	10
28.	To 4 do. as by do. ———		2	2	
May 24.	To 2 yards of dowlas, at 11 <i>d</i> .			1	10
Aug. 9.	To a parcel, as per bill —		1	15	6
			<hr/>		

This bill I cast up as a common sum in }
Addition of Money, and find the total } 48 - 2 - 8

EXAMPLE 2.

A Stationer's Bill.

Mr. *Samuel Long* bt. of *John Page*.

1783.

May 19.	2 Ream of fool's-cap superfine } paper ———	}	£.	s.	d.
June 12.	4 Ditto ———		3		
	20 Ream of coarse brown —		1	10	6
Aug. 14.	To sundry goods, as per bill		3	19	6
			<hr/>		
			£. 10	-	-

Note, Some persons never use the word *To* at the beginning of the bill. *Note* farther, the word *To* is used in books of accompts on the *Dr.* or left-hand side. The word *By* is used on the *Cr.* or right-hand side.

And now, *Tyro*, I shall draw you out a few more bills, and leave you to set them on a slate, and cast them up yourself, which will be of service, by way of exercise.

Of ADDITION.

EXAMPLE 3.

A Goldsmith's Bill.

Mr. James Proud bt. of Paul Fineshew.

1783.		£.	s.	d.
May 9.	To 1 Diamond ring	15	-	-
	2 Pair of silver salts	2	-	2 - 6
	1 Quart tankard	10	-	10 -
	1 Pint do.	4	-	7 - 6
	1 Dozen of knives and forks	3	-	2 - 6
	1 Silver tea-kettle	5	-	5 -
	12 Tea-spoons, tongs, and cream-pot	1	-	15 -

Total £.

EXAMPLE 4.

A Taylor's Bill.

Mr. Robert Patience Dr. to John Trimmer.

1783.		£.	s.	d.
Mar. 15.	To 2 Yards of cloth, at 18s. per yard	1	-	16 -
	Making your coat	-	-	8 -
	4 Yards and a half of shalloon, at 2s.	-	-	9 -
	Buckram, stay-tape, and canvas	-	-	3 - 6
	Silk, twist, and mohair	-	-	2 - 8
	Making a suit for your son	-	-	5 - 9
	Buckram, silk, twist, mohair, &c.	-	-	4 - 3
	Buttons to the same	-	-	3 - 6

£.

Now, from these four examples, *Tyro*, you may form any other tradesman's bill in due order; and as for making out or balancing any reckoning between one person and another, to see how much is due to either, that is not the work of *Addition*; but I shall explain it very fully to you in *Subtraction*.

2. *The manner or common Form of Receipts, and Notes of Hand.**Tyro*. How am I to give or write a receipt?*Philo*. According to what money you receive, and the persons you receive it of, or for. Let us take the sta-

Of ADDITION.

31

stationer's bill in example 2, which is 10*l*. Now, if he receives it in full, the receipt will run thus:

August 16, 1783. *Received of Mr. Samuel Long ten pounds in full.*

John Page.

£. 10

If the receipt be wrote upon the bill itself, then this is better:

Received at the same time the contents in full.

John Page.

EXAMPLE 2. If only part be received, then thus:

August 16, 1783. *Received of Mr. Samuel Long five pounds, on accompt.*

John Page.

£. 5

EXAMPLE 3. If the person that receives the money be a son or servant, he must write thus:

August 16, 1783. *Received of Mr. Samuel Long ten pounds in full, for my father (or master.)*

John Page, jun.

EXAMPLE 4. For Rent.

June 14, 1783. *Received of Mr. John Lumley twelve pounds, ten shillings, for half a year's rent, due at Lady-Day last.*

Abraham Gripe.

£. 12 - 10

EXAMPLE 5. When there has been an account of long standing, and at last the two parties agree to have a reckoning, but still he that owes money upon the balance has none at that time to pay, then the following form is counted better than a common note of hand, because it shews the reason of such an acknowledgment:

ledgment: but then this writing should be drawn in the book of the person to whom the balance is due, and if signed in the presence of witnesses the better.— The form is thus:

August 16, 1783. *Reckoned and balanced all accounts, and I Samuel Long acknowledge myself to be indebted to John Page three pounds, ten shillings, which I promise to pay on demand, for value received. Witness my hand,*

Test.

Samuel Long.

Abraham Justice.

EXAMPLE 6. When a person has no money about him, or has his money in other persons hands, and gives you a note or draught upon them for the payment of any sum, it is wrote after this manner:

Sir, please to pay to Mr. John Page, or bearer, three pounds, ten shillings, and place it to the account of

Your humble servant,

To Mr. Jonathan Trusty.

Samuel Long.

3. *Of the Value of the common Coins used in England, how they are expressed, and how set down.*

A *Port*, or *Portugal* piece, is set down 1*l.* 16*s.* but is expressed, or commonly called a six and thirty.

A double *Port* is 3*l.* 12*s.*

A *Moidore* is set down 1*l.* 7*s.* but is called a seven and twenty.

A *Guinea* is 1*l.* 1*s.* and expressed a guinea.

A *Crown* is expressed a crown; but set down 5*s.*

Half a *Crown* is 2*s.* 6*d.*

A *Tester* is six-pence.

A *Groat* is four-pence.

An useful EXAMPLE.

A servant laid out cash as follows: for *coals* a guinea and a half. *Cloth* three and twenty and six-pence. *Meat* seven groats. *Butter*, *eggs*, and *bacon*, nineteen-pence half-penny; and *thread* seven farthings. What was laid out in all?

Tyro.

Tyro. I set them down as follows:

	£.	s.	d.
For coals	1	11	6
Cloth	1	3	6
Meat	-	2	4
Butter, eggs, and bacon	-	1	7 $\frac{1}{2}$
Thread	-	-	1 $\frac{3}{4}$
In all	2	19	1 $\frac{3}{4}$

Philo. Very well done indeed. *Observe farther then,* that from *one* to *two* shillings, and from *one* or *two* pounds, the *expression* is different from the *setting down*. Thus *1s. 10d. $\frac{1}{2}$* is expressed two and twenty-pence halfpenny; and *1l. 19s. 6d.* is expressed nine and thirty and sixpence.

So also, though these numbers, 1300, 1753, and 2500, are properly, one thousand three hundred, one thousand seven hundred and fifty-three, and two thousand five hundred; yet they are thus expressed, thirteen hundred, seventeen hundred and fifty-three, and five and twenty hundred.

Tyro. I heartily thank you, kind sir, and if it were not too troublesome, I could wish you would set me a few questions by way of exercise.

Philo. You do very well to ask me, *Tyro*; but it will be better at the end of *Addition*, where I shall give you some useful examples.

SECTION VI.

AVOIRDUPOIS WEIGHT.

Tyro **P**RAY what is the use of this rule, or what are weighed by it?

Philo. Most things that are commonly dealt in, such as *grocery wares*, and also *cheese, butter, soap, candles, alum, brass, iron, copper, salt, hemp*, and all such sort of goods.

Tyro. What are the different names or denominations of the weights used in this rule?

Philo. The *greatest* denomination is a *ton*, and the *least* a *dram*. They run thus in order: *tons, hundreds, quarters, pounds, ounces, and drams*; of which is composed the following table, with the characters that stand for each denomination after them.

The TABLE.

16 Drams	} make	1 Ounce, marked thus	{	oz.
16 Ounces		1 Pound		lb.
28 Pounds		1 Quarter of an hundred		qr.
4 Quarters		1 Hundred weight		Ct.
20 Hundreds		1 Ton		T.

Tyro. Should I get this table by heart?

Philo. You may do as you please; I know some masters call it unnecessary; but if you take my advice, learn every table perfectly. 'Tis true, you may do the sums without it, if you have the table before you, but it would be a very bad thing to make an excuse, by saying you could do such a sum if you knew the rule by heart; and I have known many a lad lament the omission.

Tyro. I don't doubt it at all, and I will take your advice: please to tell me how to cast up those sums.

Philo. The same as in *Addition of Money*, only you stop (as is plain by the table) by different figures; and for your further information, I shall put over every row and denomination, the quantity you are to stop at, or do by, and shall give you one example at large as a standing rule for all that follows:

EXAMPLE 1.

(10)	(20)	(4)	(28)
Tons	C.	qrs.	lb.
25	- 14	- 1	- 15.
18	- 11	- 2	- 16
27	- 17	- 3	- 21.
46	- 14	- 1	- 17.
16	- 17	- 2	- 15
45	- 15	- 1	- 17.
16	- 17	- 3	- 14
<hr/>			
198	- 9	- 1	- 3

EXAMPLE 2.

(10)	(4)	(28)	(16)
C.	qrs.	lb.	oz.
42	- 1	- 17	- 10
17	- 2	- 19	- 14
21	- 3	- 22	- 11
17	- 2	- 17	- 12
24	- 1	- 22	- 15
65	- 3	- 17	- 13
24	- 1	- 21	- 10
<hr/>			

I begin

I begin at the *lbs.* and say, 14 and 7 is 21, and the 1 on the left hand (which is always called 10) is 31, that is 3 above 28; therefore I make a dot against the 7, and carry the odd 3 forward; saying, 3 that I carry and 5 is 8, and 10 on the left of it is 18, and 7 is 25, and 10 on the left is 35, that is 7 above 28; then I dot again, and carry 7 to the next figure, saying 7 and 1 is 8, and 2 on the left hand, which stands for 20, is 28; therefore, as there is nothing over, I only say 16 and 5 is 21, and 10 is 31, that is 3 above 28, which 3 I place under the row of pounds, and then telling my dots, I find them 4 (that is 4 qrs.) which I carry to the next row of qrs. saying 4 and 3 is 7, and 1 is 8, and 2 is 10, and 1 is 11, and 3 is 14, and 2 is 16, and 1 is 17 quarters. Now, as 4 quarters make an hundred, I ask how many *fours* I can have in 17, and find 4 *fours*, and 1 over, that is 4 hundred, 1 quarter, which one I place under the row of quarters, and carry the 4 to the hundreds, which I cast up by *twenty*, the same as in *Addition of Money*, saying 4 that I carry and 7 is 11, and 5 is 16, and 7 is 23, and 4 is 27, and 7 is 34, and 1 is 35, and 4 is 39; then I come back with the tens, saying 39 and 10 is 49, and 10 is 59, and so on, 69, 79, 89, 99, and 10 is 109, which is 5 twenties, and 9 over; that is, 5 tons, and 9 hundred over, which 9 I place under the hundreds, and carry 5 to the first row of the tons, and casting them up as whole numbers by tens, I find the first row 48, that is 8, and I carry 4 to the next row, which amounts to 19. Therefore the total is 198 *tons*, 9 *c.* 1 *qr.* 3 *lb.*

☞ This is a standing rule, *Tyro*, for all your other sums in *Addition*, which are cast up after the very same manner: those that you find done, are for your information and satisfaction; and such as are left undone, are for your practice. Are you satisfied with what I have told you?

Tyro, Sir, I am; but in example 2, I perceive you set 10 over the *hundreds* place, contrary to the table; whereas in example 1, you have set 20 over the *hundreds* place, which I own puzzles me at present.

Philo. That is for want of a little consideration, *Tyro*; for whatever *name* or *denomination* stands first (that is, whatever you add up last) is always added up like *whole numbers*, by *tens*; be they *tons*, *hundreds*, *pounds*, *shillings*, *yards*, *ells*, or any thing else, as you will see hereafter.

Tyro. I am obliged to you, sir, and desire no further instruction in *Addition*, but only the rules and examples to go by.

Philo. You shall not want for either, and they are all done after the same manner as this example before you, though by different figures: and pray take notice, all those examples that you find ready done, are not to indulge you in idleness; but the answers are inserted for your information and satisfaction; and those that are left undone are for your exercise, practice, and improvement; and though I told you in *Addition of Money* it is not customary to use cyphers; yet, if you choose it, take your own way; but you will find it better to leave them out, when there is no occasion to use them.

AVOIRDUPOIS *small Weight.*

Tyro. What is the use of this rule?

Philo. It is chiefly used by such as deal in silk, worsted, thread, &c. by *retail*, or in small quantities only; that is, from a dram to pounds: 16 drams make an ounce, and 16 ounces 1 pound.

EXAMPLE 1.		
(10)	(16)	(16)
lb.	oz.	dr.
2	- 10.	- 5
4	- 7	- 3
1	- 6	- 11.
6	- 9.	- 0
3	- 7	- 9
<hr/>		
18	- 8	- 12

EXAMPLE 2.		
(10)	(16)	(4)
lb.	oz.	grs.
3	- 14	- 2
1	- 11	- 1
3	- 09	- 0
2	- 01	- 2
2	- 00	- 3
<hr/>		

Note, *Worsted* is weighed by ounces and quarters, as in *example 2*, and no drams are used here, nor in retailing many other commodities.

Of WOOL.

Wool is weighed by the clove, stone, tod, wey, &c. as follows :

7 Pounds make 1 clove
 2 Cloves, or 14 *lb.* 1 stone.
 2 Stone, or 28 *lb.* 1 tod
 6½ Tod 1 wey
 2 Weys 1 sack
 12 Sacks 1 last.

Note, In some places 7 tod are allowed to 1 wey, and 12 score, or 240 *lb.* is called a *pack of wool*.

Note farther, A firkin of soap is 64 *lb.* a firkin of butter 56 *lb.* a clove of cheese 8 *lb.* a wey of cheese in *Essex* is 32 cloves, or 256 *lb.* a wey in some parts of *Suffolk* is the same; but in other parts of it, 42 cloves, or 336 *lb.* make a wey.

See another table of *weights and measures*, section 6.

TROY-WEIGHT.

Tyro. Of what use is *Troy-Weight*?

Philo. By this is weighed gold, silver, jewels, electuaries, &c. and liquors in general.

TABLE.

24 Grains	} make	{ 1 Pennywt. thus	{ <i>dw.</i>		
20 Pennywts.				{ 1 Ounce	{ <i>oz.</i>
12 Ounces					

Note, You need not *point* or *dot* at any row except the first, where you do by 24; the others are easy and common, as before, the pennyweights being cast up by 20, are done the same as shillings; and the ounces being done by 12, are cast up like pence, in *Addition of Money*.

(10)	(12)	(20)	(24)	(10)	(12)	(20)	(24)
lb.	oz.	dw.	gr.	lb.	oz.	dw.	gr.
42	- 11	- 14	- 18.	647	- 11	- 17	- 16
26	- 9	- 10	- 21	197	- 10	- 15	- 23
49	- 10	- 11	- 11.	494	- 11	- 19	- 14
65	- 8	- 14	- 12	165	- 9	- 15	- 21
73	- 9	- 15	- 16.	219	- 1	- 11	- 19
87	- 11	- 18	- 11	648	- 8	- 15	- 21
<hr/>				<hr/>			
347	- 2	- 5	- 17				

Note 2. That 1 lb. *avoirdupois-weight* is equal to about 14 ounces 12 pennyweights troy.

Note 3. Custom only introduced *avoirdupois-weight* and *beer measure*: for, according to the statute laws, there should be but one *weight* and one *measure* throughout the whole realm, as you may see under *dry measure*, *Note 1.* Therefore, it is evident, that from these two different weights, came the different sorts of measures, as you will more plainly see under *dry measure* and *liquor measure*.

Note farther, the Value of Gold and Silver.

<i>Gold.</i>		£.	s.	d.
That 1 Pound weight of gold is worth		48	- 0	- 0
1 Ounce	_____	4	- 0	- 0
1 Pennyweight	_____	0	- 4	- 0
1 Grain	_____	0	- 0	- 0
<i>Silver.</i>		£.	s.	d.
That 1 Pound weight of silver is worth		3	- 0	- 0
1 Ounce	_____	0	- 5	- 0
1 Pennyweight	_____	0	- 0	- 0
1 Grain about $\frac{1}{2}$ a farthing.				

APOTHECARIES WEIGHT.

Tyro. What is the use of this weight?

Philo. By it *apothecaries* mix and compound their medicines, their pound being the same as the pound troy, only differently divided, as you see in the following table.

Note, Though *apothecaries* mix their medicines by this rule, they buy and sell their drugs by *avoirdupois weight*.

20 G
3 S
8 D
12 C

(10
lb

42

17

43

15

47

64

232

Tyro.
Philo.

as coals
and oth

Pints

Quar

Pottle

Gallo

Pecks

Bushe

Coom

Quart

Pecks

Bushe

Quart

Note,

The

The TABLE.

20 Grains	}	make	1 Scruple	}	thus marked.
3 Scruples			1 Dram		
8 Drams			1 Ounce		
12 Ounces			1 Pound		

(10)	(12)	(8)	(3)	(20)		(10)	(3)	(20)
16	3	3	3	gr.		3	3	gr.
42 -	10 -	7 -	2 -	19		18 -	1 -	14
17 -	8 -	5 -	2 -	11		12 -	2 -	17
43 -	7 -	6 -	1 -	10		41 -	1 -	10
15 -	4 -	5 -	1 -	9		24 -	2 -	15
47 -	11 -	3 -	1 -	8		16 -	1 -	11
64 -	9 -	6 -	2 -	15		17 -	2 -	15
<hr/>						<hr/>		
232 -	5 -	4 -	0 -	12				

DRY-MEASURE.

Tyro. What is the use of *Dry-measure*?

Philo. By this rule are measured all dry goods, such as coals, sand, salt, fruit, oysters, wheat, barley, peas, and other grain, as appears by the two following tables.

TABLE 1.

1 Pints	}	make	1 Quart
2 Quarts			1 Pottle
4 Pottles, or 8 pints			1 Gallon
8 Gallons, or 16 pints			1 Peck
16 Pecks			1 Bushel
2 Bushels			1 Coomb
4 Coomb, or 8 bushels			1 Seam, or quarter
8 Quarters, or 40 bushels			1 Load

TABLE 2. For Coals.

1 Pecks	}	make	1 Bushel
2 Bushels			1 Quarter of a Chaldron
4 Quarters, or 36 bushels			1 Chaldron
Note, 5 Pecks			1 Bushel, water-measure.

Note

Note 1. That 33 cubic inches, and 3 fifths, make a corn pint; 268 inches, and four-fifths, a corn gallon; and 2150 inches, two-fifths, a true *Winchester* bushel according to act of parliament made in 1697, which says, that every round bushel with an even bottom that is 18 inches wide, and 8 inches deep, shall be deemed a true legal *Winchester* bushel.

(10) Loads	(5) qrs.	(8) B	(4) Pecks	(10) Chal.	(4) qrs.	(9) Bush.	(10) Chal.	(36) B.	(4) Pecks
24	- 4	- 7	- 3	64	- 2	- 8	47	- 31	- 2
19	- 3	- 5	- 2	17	- 1	- 6	19	- 17	- 1
49	- 2	- 3	- 1	48	- 2	- 3	47	- 16	- 2
47	- 3	- 1	- 2	17	- 1	- 4	56	- 14	- 1
56	- 1	- 5	- 3	96	- 2	- 5	17	- 13	- 2
17	- 3	- 4	- 2	17	- 3	- 7	47	- 27	- 3
<hr/>									
215	- 4	- 4	- 1						

Note 2. The common received contents of a corn gallon is 274 inches. For dry measure is a mean, as it were, between wine and beer measure. For as 12 oz. troy is to 224 inches; so is 14 $\frac{1}{2}$ oz. to 272 inches nearly. See note 3, in wine measure.

Note 3. That salt, sea-coal, and many other commodities, are heaped in the measuring in general; and where they are not, it is customary to allow 5 struck pecks to the bushel. Bran is double measure; that is, 2 pecks struck are allowed for one peck.

Observe farther.

A score of coals — is 21 chaldron, that is, those that buy 20 chaldron have 21 for 20

A sack of coals	—	3	} Bushels.
A sack of corn in common	—	4	
A sack of flour	—	5	
A load, called a market load	—	5	
A load in general means	—	40	
A wey is 10 quarters, or 2 loads	—	80	}
A last in some places is 12 wey, viz.	—	960	

See section 6, in Addition.

LIQUID

LIQUID-MEASURE.

Tyro. What is the use of this *measure*?

Philo. All sorts of wine, spirituous liquors, and beer and ale, are measured by it, under the names of *Wine measure* and *Winchester measure*.

I. Of WINE-MEASURE.

T A B L E.

6) (4) 2 Pints	}	make <	1 Quart
4 Quarts			1 Gallon
1 - 2 10 Gallons			1 Anker of brandy, or rum
7 - 1 18 Gallons			1 Runlet
6 - 2 31½ Gallons			1 Barrel of wine or vinegar
4 - 1 42 Gallons	}	}	1 Tierce
3 - 2 63 Gallons			1 Hogshead
7 - 3 2 Hhds. or 126 gallons			1 Butt, or pipe
2 Pipes, or 252 gallons			1 Tun

Note 1. A *puncheon* is 8 ankers, or 80 gallons; but any cask between a *hoghead* and a *pipe*, is called a *puncheon*.

(10) (2) (2) (63)	(10) (63) (8)
Tuns Pipes Hhds Gall.	Hhds. Gall. Pints.
47 - 1 - 1 - 21.	47 - 19 - 7
49 - 0 - 1 - 57	26 - 14 - 1
64 - 1 - 0 - 16.	94 - 17 - 5
45 - 1 - 1 - 18	57 - 47 - 6
27 - 0 - 0 - 15	19 - 14 - 4
56 - 1 - 1 - 17	45 - 49 - 5
291 - 1 - 0 - 18	

Note 2. Cyder, perry, oil, vinegar, &c. are bought and sold by this measure; and milk is also sold in the city of *London* by it; there being no standard to the contrary, corrupt custom has reduced the largest of liquid pints to one half its proper quantity.

Note 3. The *wine-pint* (according to custom) is reckoned to contain 28 cubic inches, and 7 eights; and the gallon 231 inches. But by an experiment made at *Guild-Hall*, in *London*, (1688) a vessel containing but 224 cubic inches, was filled with water, and carefully emptied into the *wine gallon* kept there, which did exactly fill it. Notwithstanding this, it was thought proper to continue 231 inches to the gallon, which remains to this day. And from this increase of the *wine gallon* (answering to *troy weight*) came the increase of the *beer gallon*, which answers to *avairdupois-weight*. See *note 1.* in *Winchester measure*.

2. WINCHESTER-MEASURE

Tyro. What are the different measures and denominations for *beer* and *ale*?

Philo. They are as under :

T A B L E.

2 Pints	}	make	1 Quart
4 Quarts			1 Gallon
9 Gallons			1 Firkin
2 Firkins, or 18 gallons			1 Kilderkin
2 Kilderkins, or 36 gallons			1 Barrel
1 $\frac{1}{2}$ Barrel, or 54 gallons			1 Hogshead
2 Hhds. or 3 bar. or 108 gall.			1 Butt
2 Butts, or 216 gallons			1 Tun

Note 1. That 35 cubic inches 1 quarter make a *beer* pint, and (nearly) 1 gallon, which answers to *avoirdupois weight*. For 12, the ounces in 1 lb. *troy*, is to 231, the inches in a *wine* gallon so is $14\frac{1}{2}$ ounces to 282, the inches in a customary *beer* gallon. See *note 2, Troy weight*.

(10)	(2)	(1 $\frac{1}{2}$)	(2)	(2)	(9)	(10)	(3)	(36)
Butts	Hhds.	Bar.	Kild	Firk.	Gall.	Butts	Bar.	Gall.
15	-	1	-	0	-	1	-	5
23	-	0	-	1	-	0	-	2
41	-	1	-	1	-	1	-	3
22	-	1	-	0	-	1	-	0
7	-	1	-	1	-	0	-	5
1	-	0	-	1	-	1	-	0
113	-	0	-	1	-	1	-	0

Note 2. That in gauging *beer* and *ale* in *London*, 32 gallons is a barrel of *ale*, and 36 a barrel of *beer*; but in other places 34 gallons is a barrel, one with another, and common brewers in the country allow victuallers 36 gallons of both sorts to the barrel.

CLOTH-MEASURE.

Tyro. What are the denominations of this measure?

Philo. Yards, quarters, and nails: also ells *English*, ells *Flemish*, and *French* ells, as appears by the table.

T A B L E.

2 Inches and a quarter	}	make	1 Nail
4 Nails			1 Quarter of a yard
4 Quarters			1 Yard
3 Quarters of a yard			1 Ell <i>Flemish</i> measure
5 Quarters, or 1 yd. 1 qr.			1 Ell <i>English</i>
6 Quarters			1 French ell

Note 1. That things in common, such as *woollen, linen, silk, tape, cord, &c.* are measured by the yard; but *hollands* are in general measured by the ell *English*, and *tapestry* by the ell *Flemish*.

Note 2. That 16 nails make a yard, 20 nails an ell *English*, and 12 nails an ell *Flemish*.

(10) (4) (4)	(10) (5) (4)	(10) (3) (4)
Yds. qrs. Nails.	Ells Eng. qrs. N.	Ells Fl. qrs. N.
47 - 3 - 2	64 - 4 - 3	17 - 2 - 3
19 - 2 - 1	17 - 2 - 1	41 - 1 - 2
16 - 3 - 2	19 - 3 - 2	19 - 2 - 1
14 - 1 - 1	15 - 4 - 3	64 - 1 - 2
17 - 1 - 2	16 - 2 - 1	14 - 2 - 1
21 - 2 - 1	17 - 2 - 3	25 - 1 - 3
37 - 2 - 1		

LONG-MEASURE.

Tyro. What does *long measure* teach?

Philo. To know the length or breadth of any thing, and the distance of one thing, or place, from another, by the following table.

T A B L E.

3 Barley-corns	}	make	1 Inch
12 Inches			1 Foot
3 Feet			1 Yard
2 Yards, or 6 feet			1 Fathom
$\frac{1}{2}$ Yards			1 Rod, pole or perch
10 Rods			1 Furlong
8 Furlongs			1 Mile
3 Miles			1 League
30 leagues, or 60 miles			1 Degree
60 Degrees			The circumference of the earth and sea.

Note 1. An hand, or hand's breadth, in horsemanship, is 4 inches.
A common pace is 2 feet 6 inches.
A geometrical pace is 5 feet.

2. WINCHESTER-MEASURE

Tyro. What are the different measures and denominations for *beer* and *ale*?

Philo. They are as under :

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9 Gallons			1 Firkin
2 Firkins, or 18 gallons			1 Kilderkin
2 Kilderkins, or 36 gallons			1 Barrel
1 $\frac{1}{2}$ Barrel, or 54 gallons			1 Hoghead
2 Hhds. or 3 bar. or 108 gall.	}		1 Butt
2 Butts, or 216 gallons			1 Tun

Note 1. That 35 cubic inches 1 quarter make a *beer* pint, and a (nearly) 1 gallon, which answers to *avoirdupois weight*. For 12, the ounces in 1 *lb. troy*, is to 231, the inches in a *wine* gallon so is $14\frac{1}{2}$ ounces to 282, the inches in a customary *beer* gallon. See *note 2, Troy weight*.

(10)	(2)	(1 $\frac{1}{2}$)	(2)	(2)	(9)	(10)	(3)	(36)
Butts	Hhds.	Bar.	Kild	Firk.	Gall.	Butts	Bar.	Gall.
15	-	1	-	0	-	1	-	5
23	-	0	-	1	-	0	-	2
41	-	1	-	1	-	1	-	3
22	-	1	-	0	-	1	-	2
7	-	1	-	1	-	0	-	5
1	-	0	-	1	-	1	-	4
<hr/>						<hr/>		
113	-	0	-	1	-	1	-	3

Note 2. That in gauging *beer* and *ale* in *London*, 32 gallons barrel of *ale*, and 36 a barrel of *beer*; but in other places 34 gallons is a barrel, one with another, and common brewers in the country allow victuallers 36 gallons of both sorts to the barrel.

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4 Quarters			1 Yard
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6 Quarters			1 French ell

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Note 2. That 16 nails make a yard, 20 nails an ell *English*, and 12 nails an ell *Flemish*.

(10) (4) (4)	(10) (5) (4)	(10) (3) (4)
<i>Yds. qrs. Nails.</i>	<i>Ells Eng. qrs. N.</i>	<i>Ells Fl. qrs. N.</i>
47 - 3 - 2	64 - 4 - 3	17 - 2 - 3
19 - 2 - 1	17 - 2 - 1	41 - 1 - 2
16 - 3 - 2	19 - 3 - 2	19 - 2 - 1
14 - 1 - 1	15 - 4 - 3	64 - 1 - 2
17 - 1 - 2	16 - 2 - 1	14 - 2 - 1
21 - 2 - 1	17 - 2 - 3	25 - 1 - 3
<hr/>	<hr/>	<hr/>
37 - 2 - 1		

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T A B L E.

3 Barley-corns	}	make	1 Inch
12 Inches			1 Foot
3 Feet			1 Yard
2 Yards, or 6 feet			1 Fathom
$\frac{1}{2}$ Yards			1 Rod, pole or perch
40 Rods			1 Furlong
8 Furlongs			1 Mile
3 Miles			1 League
30 Leagues, or 60 miles			1 Degree
60 Degrees			The circumference of the earth and sea.

Note 1. An hand, or hand's breadth, in horsemanship, is 4 inches.
A common pace is 2 feet 6 inches.
A geometrical pace is 5 feet.

(10)	(20)	(3)	(8)	(40)	(10)	(5 $\frac{1}{2}$)	(3)	(12)
Deg.	Leag.	Miles	Fur.	Rods	Rods	Yds.	Feet	Inches
25	- 19	- 2	- 7	- 21.	42	- 4	- 1	- 11
17	- 14	- 1	- 5	- 19	61	- 3	- 0	- 10
16	- 8	- 2	- 2	- 15.	17	- 1	- 1	- 9
21	- 5	- 1	- 4	- 27	24	- 0	- 2	- 8
47	- 7	- 2	- 1	- 14.	16	- 2	- 1	- 7
15	- 10	- 1	- 5	- 37	72	- 1	- 0	- 6

144 - 7 - 0 - 3 - 13

Note 2. According to the table, 60 miles make one degree, therefore the earth is 21600 miles round: but 69 $\frac{1}{2}$ miles (very near) make but one degree, and therefore the circumference of the earth is about 25000 miles; as you will see in *Reduction*.

LAND-MEASURE.

Tyro. What are the denominations of *land measure*?

Philo. Almost the same as in *long measure*; but they never regard the inches and barley-corns, the table is sufficient.

T A B L E.

5 $\frac{1}{2}$ Yards, or 16 feet $\frac{1}{2}$	} make {	1 Rod, pole or perch
40 Rods or poles		1 Furlong in length
40 Rods in length, and 1 in breadth		1 Rood or quarter of an acre
4 Roods, or quarters		1 Acre

Note 1. That though 16 $\frac{1}{2}$ feet make a statute pole or rod, yet it is customary in some low, fenny countries, and barren lands to allow 18 feet, and in measuring forests 21 feet to the pole.

(10)	(4)	(40)	(5 $\frac{1}{2}$)	(10)	(4)	(40)
Acres	Roods	Poles	Yds.	Acres	Roods	Poles
7	- 1	- 14.	- 1	15	- 1	- 19
9	- 3	- 27	- 2	26	- 2	- 24
4	- 1	- 29.	- 3	19	- 3	- 27
15	- 2	- 27	- 4	47	- 1	- 15
37 - 1 - 18 - 4 $\frac{1}{2}$						

Note 2. The common instrument used in measuring land is iron chain, containing 100 links, which is 4 rods, or 22 yards length; therefore 10 chains in length, and 1 in breadth, make an acre, and 80 chains in length only make a mile, or 1760 yards.

S Q U A R E

Chain 22 yds.

Tyro.
Philo.
the natur
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you the c
notion of
be amifs.
measure) t
by and
16 Squa
144 Squa
9 Squa
10 Squa
272 sq
100 Squa
Note,
Tyro.
Philo.
changin
things.
minutes,
serve all

S Q U A R E - M E A S U R E .

Tyro. What do you mean by *square measure*?

Philo. You are not to expect, *Tyro*, as yet to know the nature of it; it is sufficient, at present, that you only know this, that *long measure* shews you only the length or breadth of any thing; but *square measure* tells you the content of any thing, which you can have no notion of till you have learnt *Division*; but it will not be amiss to learn the following table by heart (at your leisure) that you may be the better able to understand it by and by.

16 Square quarters	}	make	{	1 Square inch
144 Square inches				1 Square foot
9 Square feet				1 Square yard
40 Square yards, 1 qr. or				1 Square rod, or
272 square feet 1 quarter				pole
160 Square rods				1 Acre of ground

Note, An example or two will be sufficient.

(10) (9)
Yards Feet

45 - 6

29 - 3

47 - 8

25 - 5

148 - 4

(10) (9) (144) (16)
Yards Feet Inches Qrs.

17 - 5 - 71 - 3

64 - 2 - 14 - 6

16 - 4 - 17 - 12

78 - 5 - 94 - 11

Of T I M E .

Tyro. What is *time*, and how is it divided?

Philo. *Time* shews us the beginning, mutation, (or changing) continuation, and ending of all mutable things. It is measured by *years*, *months*, *days*, *hours*, *minutes*, and *seconds*, and divided as follows, which will serve all common purposes.

TABLE

T A B L E.

60" Seconds	}	make	{	1 Minute of time
60' Minutes				1 Hour
24 Hours				1 Natural or real day
7 Days				1 Week
4 Weeks				1 Month
13 Months, or 365 Days				1 Year

Note 1. Though the table says 13 months, or 365 days, make a year, yet it is not truly so; for 13 months (allowing four weeks to the month) is but 364 days, whereas 13 months, 1 day, 6 hours, make a year; for these odd 6 hours make 24 hours, or 1 day, every fourth year, which is added to *February*, which has then 29 days, and is called *leap year*: but a true year is 365 days, 5 hours, 48 minutes, and is called a *sidereal year*, being the time that the sun performs its apparent revolution through the *ecliptic*; but you must not trouble yourself, and lose your time about these things, which are too hard for you to understand at present.

Note 2. Though 13 months are said to make a year, and servants commonly reckon a month 28 days, yet you are to observe, that in trade and transacting business, by a month is meant a *calendar month*; that is, from any day of the month to the same day of the next month: thus, from the 5th of *February* to the 5th of *March*, or from the 18th of *April* to the 18th of *May*, is a month.

EXAMPLE 1.

(10) Yrs.	(13) Mths.	(4) Wks.	(7) Days
27	- 11	- 3	- 6
43	- 9	- 2	- 5
27	- 5	- 1	- 4
36	- 1	- 2	- 3
45	- 10	- 2	- 1
28	- 12	- 1	- 4
<hr/>			
209	- 12	- 2	- 2

EXAMPLE 2.

(10) Days	(24) Hrs.	(60) Min.	(60) Sec.
41	- 22	- 50	- 27
17	- 17	- 17	- 15
24	- 15	- 27	- 14
19	- 21	- 19	- 24
27	- 11	- 18	- 19
35	- 14	- 25	- 25
<hr/>			

Note 3. In example 2, you must point or dot at every row but the last.

And now, Tyro, I think you may by this time be perfect in Addition, I shall only set you a few questions, and proceed to Subtraction.

SECTION VII.

Containing some other useful Things necessary to be known in Number, Weight, and Measure.

Note 1. Of Things bought or sold by the dozen, score, or gross.

A Dozen is 12. A score is 20. A common hundred is 100. A long hundred is 120. A gross is 12 dozen, or 144, and a great gross is 12 times as many, or 1728. Apples, oranges and lemons, corks, bungs, and many other things, are bought and sold by the dozen or gross. Herrings, and several other sorts of fish, and all sorts of nails, and many such small things, have six score, or 120, to the hundred; but a hundred of ling cod is 124 in number; and a hundred books in printing is 104.

Note 2. Of PARCHMENT and PAPER.

A dozen is 12 skins; 5 dozen one roll of parchment; and sometimes 25 sheets make a quire of paper, 20 quires 1 ream, and 10 ream 1 bale.

Note 3. Of the different Sizes of BOOKS.

Folio is the largest of all books, and has but two leaves to the sheet. Quarto (marked 4to) has four leaves to a sheet. Octavo (or 8vo) is a sheet doubled into 8 parts, and duodecimo (commonly called twelves, and marked 12mo) has 12 leaves to the sheet.

Note 4. Of WEIGHT, MEASURE, &c.

A faggot of steel 6 score, or 120 lb. A burthen or load of steel 9 score. A barrel of anchovies 30 lb. A barrel of figs from 98 to 300 lb. A barrel of gunpowder 1 Cwt.

1 Cwt. A *puncheon* of prunes from 10 to 12 Cwt. *ton* or *fother* of lead 19 Cwt. 2 qrs. A *quintal* of 100 in tale. A stone of iron, shot, or horseman's weight 14 lb. A stone of meal 8 lb. A stone of hemp 32 lb. A stone of glass 5 lb. A *seam* of glass 24 stone, or 120 lb. A *keg* of herrings, &c. 60 in number, an hundred is 120. A *cade* of sprats 1000. A *cade* of herrings 500. A *barrel* 1000. A *last* 12 barrels, or 12,000. A *last* of rape-seed 10 qrs. A *last* of gunpowder 24 barrels. A *last* of leather 20 dickers. A *dicker* 10 skins. A *last* of hides 12 dozen; of tar 14 barrels; of wool 12 facks. A *wey* of flax or feathers 1700 lb. A *wey* in some places is a chaldron. A *wey* of meal 6 qrs. A gallon of train oil 7½ lb. A *tun* 252 gallons. A *tun* of sweet oil 236 gallons. A *load* of hay in some places is 25, in others 30 cwt. In London it is sold in *trusses*, containing 56, 60 lb. a truss, and 36 trusses to the load. A *load* of Scotch coals 1 cwt. A load of tiles 1000. Of bricks 500. Bricks in general are 9 inches long, 4½ inches broad, and 2½ thick. A *square* of tiling or thatching contains 10 feet every way, that is 100 feet, and a *load* of brick work 272 feet, 1 quarter; but 272 is reckoned for common work. A *stack* of wood is 3 feet high, 3 feet wide, and 12 feet long; but this is according to the agreement of the master and the workmen.

Tyro. I am extremely obliged to you, Sir; but pray am I bound to get these things by heart before I learn *subtraction*?

Philo. Not at all, *Tyro*: to learn the rules and the common tables is sufficient. The others I have only added for your further instruction: They are not set you a task, but for your diversion; and if you now and then read them over, you will soon find the benefit of them in improving your mind; since it will naturally gain you the good-will of your parents, your master, and mankind in general, rather than squandering away your time in idleness and mischief, besides the disgrace of living and dying a dunce.

SECTION VIII.

Containing some useful and diverting Questions to exercise the Learner in ADDITION only.

Quest. 1. **A** Man borrowed of his friend a certain sum of money, and paid him in part 5*l.* 10*s.* and left unpaid 24*l.* 10*s.* What did he borrow? *Ans.* 40*l.*

Rule. Adding the two sums together gives the answer.

2. Suppose a person was born in 1709, when will he be fourscore years old? *Ans.* in 1789.

Rule. Add as many to 1709 as will make it 1789, is the answer.

3. *A*, *B* and *C* agreed to purchase an estate. *A* laid out, or paid for his part, 140*l.* 10*s.* *B* paid 217*l.* 10*s.* *C* paid 500*l.* 10*s.* What did the estate cost? *Ans.* 858*l.* 10*s.*

Rule. Add all the sums together.

4. A factor bought 4 bags of hops; the first (No. 1.) weighed 2 C. 1 qr. 14 lb. No. 2. 3 qrs. 17 lb. No. 3. 1 C. 3 qrs. 13 lb. And No. 4. weighed 1 qr. 27 lb. What is the weight of all? *Ans.* 6 C. 2 qrs. 15 lb. All together is the answer.

5. A shopkeeper having opened a shop, sold the first day as many goods as came to 15*l.* 13*s.* 7*d.* $\frac{3}{4}$, and thus went on for one week, (*viz.* 6 days.) How much he take in all? *Ans.* 94*l.* 1*s.* 10*d.* $\frac{1}{2}$.

Rule. Set down 15. 13. 7. $\frac{3}{4}$. six times, and adding them together gives the answer.

6. A farmer carried out 100*l.* in order to buy cattle, and brought home but 24*l.* 15*s.* 6*d.* What did he lay out? *Ans.* 65*l.* 4*s.* 6*d.* For these two sums added together make just 100*l.* and so on for any other sum.

DIALOGUE III.

SECTION I.

SUBTRACTION in whole Numbers.

Tyro. **W**HAT does subtraction teach?

Philo. Subtraction teaches to take a less number from a greater to discover the difference. To prove the work, add the difference to the less number, and if that sum be the same as the greater, the work is right, otherwise false.

Tyro. How is subtraction performed?

Philo. Quite contrary to addition; for in addition you use the word *and*, but here you use the word *from*. Thus 4 from 7, that is, 4 taken out of 7, there remains 3 for the difference. Again, 5 from 10, there remains 5, and 6 from 14, there remains 8, and so for any other numbers, as you may see by the following examples:

	EXAMPLE 1.	Ex. 2.	Ex. 3.	Ex. 4.
	£.	Yards.	Ells.	lbs.
Greater number	27	95	468	441756
Lesser number	14	61	162	240256
	—	—	—	—
Difference	13	34	306	201500
	—	—	—	—
Proof	27	95		

The WORK in Words at Length.

In Example 1, I say, 4 from 7 there remains 3; and from 2 there remains 1. To prove it, I add the difference to the less number, saying, 3 and 4 is seven, and 1 and 2 makes 2. In Ex. 2, I say, 1 from 5 there remains 4, and 6 from 9 there remains 3. Proof, I say, 4 and 1 is 5, and 6 is 9: That is, 34 and 61 added together makes 95, and so for the other examples.

Tyro. This is quite plain indeed.

Philo. I shall work examples 3 and 4 in the same manner, and leave you to prove them as I have directed.

Obse

Of SUBTRACTION.

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Observe then, *Ex. 3*, I say, 2 from 8 there remains 6; then 6 from 6, or 6 taken out of 6, there remains 0, and 1 from 4 there remains 3. *Lastly*, in *Ex. 4*, I say, 6 from 6 there remains 0; then 5 from 5 there remains 0; 2 from 7 there remains 5; then 0 from 1, that is nothing taken out of 1, there still remains 1; then 4 from 4 there remains 0, and 2 from 4, 2.

Note, I shall give you an example or two with, and without cyphers, that you may endeavour to avoid that idle custom.

<i>Ex. 5, with cyph.</i>	<i>without.</i>	<i>Ex. 6, with cyph.</i>	<i>without.</i>
From 24526	24526	84257	84257
Take 00012	12	84215	84215
<u>Ans. 24514</u>	<u>24514</u>	<u>00042</u>	<u>42</u>

Tyro. I own that the examples without the cyphers look best, and I understand very well what you have shewn me.

Philo. I shall leave you then 3 or 4 examples for practice.

<i>Yards</i>	<i>Ells</i>	<i>Hundreds</i>
From 47162	94785	347621
Take 12412	3104	301

Tyro. Now, sir, be pleased to shew me how to subtract or manage a sum, when the lower figure is sometimes larger than the top one; for I think that it appears difficult to me at present.

Philo. Never fear; you will soon find it easy, if you do but observe the following rule.

When the lower figure is larger than the top one, the rule is,

Take the lower figure out of what you do by, which (in whole numbers) you know is ten, and to that difference, add also the top figure, and that is the difference, which place under the first row. This is what is called borrowing in subtraction; therefore remember that you are always to carry 1 to the next lower figure for so doing: an example or two will make quite plain.

DIALOGUE III.

SECTION I.

SUBTRACTION in whole Numbers.

Tyro. WHAT does *subtraction* teach?

Philo. *Subtraction* teaches to take a *less number* from a *greater* to discover the *difference*. To prove the work, add the *difference* to the *less number*, and if that sum be the same as the *greater*, the work is right, otherwise false.

Tyro. How is *subtraction* performed?

Philo. Quite contrary to *addition*; for in *addition* you use the word *and*, but *here* you use the word *from*. Thus 4 *from* 7, that is, 4 *taken out of* 7, there remains 3 for the *difference*. Again, 5 *from* 10, there remains 5, and 6 *from* 14, there remains 8, and so for any other numbers, as you may see by the following examples:

	EXAMPLE 1.	Ex. 2.	Ex. 3.	Ex. 4.
	£.	Yards.	Ells.	lbs.
Greater number	27	95	468	441756
Lesser number	14	61	162	240256
	—	—	—	—
Difference	13	34	306	201500
	—	—	—	—
Proof	27	95		

The WORK in Words at Length.

In *Example 1*, I say, 4 *from* 7 there remains 3; and *from* 2 there remains 1. To prove it, I add the *difference* to the *less number*, saying, 3 and 4 is seven, and 1 and 2 makes 3. In *Ex. 2*, I say, 1 *from* 5 there remains 4, and 6 *from* 9 there remains 3. *Proof*, I say, 4 and 1 is 5, and 3 and 6 is 9: That is, 34 and 61 added together makes 95, and so for the other examples.

Tyro. This is quite plain indeed.

Philo. I shall work *examples 3 and 4* in the same manner, and leave you to prove them as I have directed.

Obse

Of SUBTRACTION.

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Observe then, *Ex. 3*, I say, 2 from 8 there remains 6; then 6 from 6, or 6 taken out of 6, there remains 0, and 1 from 4 there remains 3. *Lastly*, in *Ex. 4*, I say, 6 from 6 there remains 0; then 5 from 5 there remains 0; 2 from 7 there remains 5; then 0 from 1, that is nothing taken out of 1, there still remains 1; then 4 from 4 there remains 0, and 2 from 4, 2.

Note, I shall give you an example or two with, and without cyphers, that you may endeavour to avoid that idle custom.

<i>Ex. 5, with cyph.</i>	<i>without.</i>	<i>Ex. 6, with cyph.</i>	<i>without.</i>
From 24526	24526	84257	84257
Take 00012	12	84215	84215
<u>Ans. 24514</u>	<u>24514</u>	<u>00042</u>	<u>42</u>

Tyro. I own that the examples without the cyphers look best, and I understand very well what you have shewn me.

Philo. I shall leave you then 3 or 4 examples for practice.

	<i>Yards</i>	<i>Ells</i>	<i>Hundreds</i>
From	47162	94785	347621
Take	12412	3104	301
	<u> </u>	<u> </u>	<u> </u>

Tyro. Now, sir, be pleased to shew me how to subtract or manage a sum, when the lower figure is sometimes larger than the top one; for I think that it appears difficult to me at present.

Philo. Never fear; you will soon find it easy, if you do but observe the following rule.

When the lower figure is larger than the top one, the rule is,

Take the lower figure out of what you do by, which (in whole numbers) you know is ten, and to that difference, add also the top figure, and that is the remainder, which place under the first row. This is what is called borrowing in subtraction; therefore remember that you are always to carry 1 to the next lower figure for so doing: an example or two will make quite plain.

	Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
	£.	Ells	Yards	Busbels
From	762	85420	760410	5217624
Take	145	17273	457146	3471276
	<hr/>	<hr/>	<hr/>	<hr/>
Difference	617	68147		

Here, in *Ex. 1.*, I say, 5 out of 2 I cannot have; therefore I take 5 out of 10, (which is what I do by here) and there remains 5, and the top figure 2 makes 7, which I place under the first row, and carry 1 to the next lower figure; saying, 1 that I carry to 4 makes 5, which taken out of 6, that is, 5 from 6 there remains 1; but now I carry nothing to the next figure, because the lower figure being less than the top one, I had no occasion to borrow; therefore I only say in the last row, 1 from 7 there remains 6. Again, in *Ex. 2.*, I say, 3 from 0 I can't, but 3 from 10 there remains 7, and 0 is 7 still; then I carry 1 to 7 is 8 from 2 I can't, but 8 from 10 there remains 2, and the top figure makes 4: then I carry 1 to 2, which is 3, saying from 4 there remains 1; but now I do not carry any to the next figure, because I did not borrow, but only say, 7 from 5 I can't take, but 7 from 10 there remains 3, and the top 5 makes 8. Lastly, I carry 1 to 1 is 2 from 8 there remains 6.

Tyro. This is plain enough.

Philo. This is the easiest way for a learner, but there is another method which is more practicable and expeditious, if you mind and learn it.

Another way to subtract, when the lower figure is larger than the top one.

When you cannot take the lower figure from the top one, then count the top figure ten more than really is. Thus, if the top figure be 2, call it 12; if 3, call it 13; if 5, call it 15; if 8, call it 18, &c. then take the lower figure from it, and you have the true answer or difference; but always remember to carry 1 to the next figure for so doing. This is called borrowing ten, and your carrying 1 to the next figure is paying of it again.

Of SUBTRACTION.

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EXAMPLES.

	£.	Yards	Ells
From	75043	41725	172560
Take	27365	17258	87275
	<hr/>	<hr/>	<hr/>
Difference	47678	24467	-----
	<hr/>	<hr/>	
Proof	75043	41725	

First, I say, 5 from 3 I can't; but calling 3, 13, I say 5 from 13, there remains 8; then I carry 1 to 6 is 7 from 4 I can't, but I say 7 from 14 there remains 7; then I carry 1 to 3 is 4 from 0 I can't, but 4 from 10 there remains 6. Again, I carry 1 to 7 is 8 from 5 I can't, but 8 from 15 there remains 7. *Lastly*, I carry 1 to 2 is 3 from 7 and there remains 4.

Note, Tho' the bottom figures in *subtraction* may be larger than the top ones; yet you are to remember the last lower figure is never larger than the top one when there is an equal number of figures.

Tyro. I understand you very well, sir.

Philo. Then I shall only leave you an example or two, to try as you have leisure.

MORE EXAMPLES.

	£.	lb.	Miles
From	417215	62170071	417621700
Take	241729	7210943	631720
	<hr/>	<hr/>	<hr/>
Difference			

SECTION II.

Of MONEY.

Tyro. **H**OW is *subtraction* of money performed?

Philo. By taking or subtracting every denomination in the lower line out of or from the upper sum, as will appear more plain by the 2 following rules.

Rule 1. When the lower figures in the *farthings*, *pence*, or *shillings*, are smaller than the top ones; then only subtract or take one from the other, and the re-

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mainder is the true *answer*, or *difference*, which is placed under each row to which it belongs.

Rule 2. When the lower figures in the *farthings*, *pence*, or *shillings*, are larger than the top ones; then subtract or take the lower figure out of what you do by; that is, take the lower *farthings* out of 4, the lower *pence* out of 12, and the lower *shillings* out of 20, taking in the top figure besides; so shall this be the true difference or answer; but remember, that you are always to carry 1 to the next figure for borrowing, as you did in *whole numbers*.

Tyro. Please to give me an example or two, I shall soon understand it.

Philo. You cannot miss if you mind the rule well.

	Ex. 1.			Ex. 2.			Ex. 3.		
	£.	s.	d.	£.	s.	d.	£.	s.	d.
From	9	8	11	48	15	5	643	14	6 $\frac{1}{2}$
Take	7	5	6	27	11	3	231	11	2 $\frac{1}{4}$
	<hr/>			<hr/>			<hr/>		
Rem.	2	3	5	21	4	2	412	3	4 $\frac{1}{2}$
	<hr/>			<hr/>			<hr/>		
Proof	9	8	11	48	15	5	643	14	6 $\frac{1}{2}$

1. I begin at the pence in example 1, saying, 6 from 11 pence there remains 5 pence, which I place under the pence; then I say, 5 shillings from 8 there remains 3 shillings; and 7 pounds from 9 there remains 2. So the *difference* is 2*l.* 3*s.* 5*d.*

P R O O F.

The work is proved like *whole numbers*, by adding the *difference* to the *lower* or *lesser number*: thus, 5 pence and 6 is 11 pence; then 3 shillings and 5 make 8 shillings; and two added to 7 make 9 pounds.

In example 2, I say, 3 from 5 there remains 2 pence; then 11 from 15 there remains 4; then 7 from 8 there remains 1; and 2 from 4 there remains 2. *Ex. 3.* having farthings, I say, 1 farthing from 3 there remains 2 farthings, which I set down thus $\frac{1}{2}$; then 2 from 6 there remains 4 pence; then 11 from 14 shillings there remains 3. *Lastly*, for the pounds, I say, 1 from 3 there remains

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Of SUBTRACTION.

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remains 2; then 3 from 4 there remains 1; and 2 from 6 there remains 4: so is the *difference* or *answer* 412l. 3s. 4d. $\frac{1}{2}$. Now if you add 412l. 3s. 4d. $\frac{1}{2}$ to the lower sum 231l. 11s. 2d. $\frac{1}{4}$, you will find they amount to the same as the top sum, *viz.* 643l. 14s 6d. $\frac{3}{4}$.

Note, After this manner is every sum done and proved, in *subtraction*.

More Examples for Practice.

From	471 - 11 - 2 $\frac{1}{4}$	576 - 19 - 11 $\frac{1}{2}$	409 - 11 - 5 $\frac{3}{4}$
Take	120 - 10 - 2	132 - 13 - 4 $\frac{1}{4}$	304 - 11 - 3 $\frac{1}{2}$
	<hr/>	<hr/>	<hr/>

Rem.

2. *When the shillings, pence, and farthings, are larger in the lower line than in the top one.*

EXAMPLE 4. EXAMPLE 5. EXAMPLE 6.

	(10)	(20)	(12)	(10)	(20)	(12)	(10)	(20)	(12)	(4)
	£.	s.	d.	£.	s.	d.	£.	s.	d.	
Lent	10	-	4 - 6	42	-	11 - 6	453	-	13 - 5	$\frac{1}{4}$
Received	7	-	10 - 8	17	-	14 - 10	326	-	15 - 8	$\frac{3}{4}$
	<hr/>		<hr/>	<hr/>		<hr/>	<hr/>		<hr/>	
Due	2	-	13 - 10	24	-	16 - 8	126	-	17 - 8	$\frac{1}{2}$
	<hr/>		<hr/>	<hr/>		<hr/>	<hr/>		<hr/>	
Proof	10	-	4 - 6	42	-	11 - 6	453	-	13 - 5	$\frac{1}{4}$

Here I say, 8 pence from 6 pence I can't, but 8 from 12 (which is what I do by at pence) there remains 4, and the top 6 makes 10: then I carry 1, because I borrowed, saying, 1 that I carry to 10 is 11 shillings, which from 4 I can't take, but 11 from 20 (which is what I do by) there remains 9, and the top 4 makes 13. *Lastly*, I carry 1 to 7 is 8 from 10 there remains 2.

P R O O F.

I prove these examples the same way as before, by adding the *difference* to the *lesser sum*, saying, 10 pence and 8 is 18 pence, that is, 6 pence above a shilling, or 12 pence; and I carry 1 to 13 is 14, and 10 is 24, which is 4 above 20 shillings, or 1l. *Lastly*, I carry 1 to 7 is 8, and 2 makes 10.

D 4

Now,

Now, *Tyro*, pray try at *Example 6*, where there are farthings.

Tyro. In *Example 6*, I say, first, 3 farthings from 1 farthing I can't, but 3 farthings from a penny, or 4 farthings, there remains 1, and the top farthing makes 2 farthings, which I set down thus $\frac{1}{2}$: then I carry 1 to 8 pence is 9 pence, which from 5 pence I can't have, but 9 from 12 there remains 3, and the top 5 makes 8 pence: again, I carry 1 to 15 is 16 from 13 I can't, but 16 from 20 there remains 4, and the top 13 is 17 shillings. *Lastly*, I carry 1 to 6 is 7 from 3 I can't, but 7 from 13 there remains 6; or 7 from 10 there remains 3, and the top 3 is 6: then I carry 1 to 2 is 3 from 5 there remains 2; but now carry none, because I did not borrow; therefore I only say 3 from 4 there remains 1.

P R O O F.

I prove this, as before directed, saying, 2 farthings and 3 is 5 farthings, which is 1 farthing above a penny, and I carry a penny to 8 is 9, and 8 is 17 pence, 5 above 12, and I carry 1 to 17 is 18, and 15 is 33, which is 13 above 20, and I carry 1 to 6 is 7, and 6 is 13, which is 3 above 10, and I carry 1 to 2 is 3, and 2 is 5; then I say 1 and 3 is 4.

Philo. You have done very well indeed; I shall only set you an example or two for practice.

EXAMPLES for Practice.

	£.	s.	d.		£.	s.	d.
Borrowed	217	- 10	- 0	Debtor	1000	- 00	- 0
Paid	109	- 15	- $8\frac{1}{4}$	Creditor	910	- 15	- $6\frac{1}{2}$
	<hr/>				<hr/>		
To pay				Balance			

Tyro. These I can do very well; therefore wish you would shew me how to balance an accompt, or make out a small reckoning.

Philo. That I will.

3. *Practical Questions for Business.*

	£.	s.	d.
<i>Quest. 1. A borrowed of B</i>	—	100	00 0
<i>A paid him at one time</i>	21	14	6
<i>At another 10 guineas, viz.</i>	10	10	0
<i>At another 20 guineas, viz.</i>	21	00	0
<i>At another time</i>	5	15	0
<i>Sold him goods amounting to</i>	18	14	9
	77	14	3
<i>In all</i>	77	14	3

What is still due to B? *Answer* 22 - 5 - 9

Rule. I add up together all the sums that *A* paid at different times, and find they amount to 77*l.* 14*s.* 3*d.* which I place under 100*l.* and subtracting it therefrom, I find that there is still due to *B* 22*l.* 5*s.* 9*d.* *answer.*

*Quest. 2. A Blacksmith delivered a Bill to a Farmer, of 45*l.* 14*s.* 6*d.* $\frac{1}{4}$, and the Farmer has paid him in part as under:*

	£.	s.	d.
<i>By cash</i>	20	00	0
<i>By 10 bushels of malt</i>	1	15	0
<i>By a load of hay</i>	1	16	0
<i>By meat at several times</i>	2	7	6
<i>By 14 bushels of oats</i>	2	11	0
<i>By 20 bushels of wheat</i>	4	7	3
	32	16	9
<i>Blacksmith's bill</i>	45	14	6 $\frac{1}{4}$
<i>Farmer's bill</i>	32	16	9

Balance due to the Blacksmith 12 - 17 - 9 $\frac{1}{4}$ *answer.*

Quest. 3. A steward collected as much money for rents as amounted to ——— £.4000

And remitted to his master as under : ———

By cash at one time	———	500
At another	———	500
At another	———	210
At another	———	420
By tax bills discounted	———	41 - 15 - 6 $\frac{1}{4}$
By repairs done to the estate		641 - 14 - 9
By other charges	———	15 - 13 - 6

Disbursed in all £.2329 - 3 - 9 $\frac{3}{4}$

Which taken from the top £.4000 ———
there remains due to his master 1670 - 16 - 2 $\frac{1}{4}$

Tyro. I understand you well.

Philo. Then I will leave you one *question* for practice, and you may do the work at leisure.

Quest. 4. Two persons, *A* and *B*, have a reckoning to settle as follows: *A* lent *B* £.300, and some time after lent him 100 guineas more: *B* paid him at 3 several times, each 100 guineas, and at another time gave him a *draught* or *note* upon *C* for £.50, and sold him as many goods as came to £.34-4-6: now I demand how the balance stands between them. *Ans.* there is still due to *A* £.5-15-6. To prove it.

Rule. Add the two sums that *A* lent to *B* together, and you will find them £.405; then set all that *B* paid in cash, and the *draught* and goods, all under one another, and add them together, which makes £.399-4-6: this being done, set £.399-4-6 under £.405, and *subtract* it therefrom, and you will find there remains £.5-15-6 due to *A* on balance.

Tyro. I will try it directly, and am sure it is easy enough, but your answer will be some help to me, I own.

Philo. As every accompt of *debtor* and *creditor* is settled after this manner, I shall give no more examples, expecting by this time you are capable of settling yourself questions; and if you are not, it is highly necessary

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necessary to look over the examples once more, then will you be duly qualified for those questions I shall set you at the end of *Subtraction*; and therefore we will proceed to *Weights and Measures*, where you will find some examples done for your instruction, and some left undone for your practice.

AVOIRDUPOIS-WEIGHT.

	Tons	C.	qr.	lb.		C.	qr.	lb.	oz.	dr.							
Bought	17	-	11	-	1	-	15	Bought	47	-	1	-	17	-	11	-	14
Sold	14	-	17	-	3	-	21	Sold	19	-	0	-	25	-	10	-	15
<hr/>																	
Unfold	2	-	13	-	1	-	22	Unfold									

Now, *Tyro*, I shall shew you this first *example*, but no more; because all other sums in every one of the rules are done in the same manner, and there is no occasion for further instruction.

First, I say, 21 from 15 I can't, but 21 from 28 (which is what I do by at *lbs.*) there remains 7, and 15 makes 22: then I carry 1 to 3 is 4 from 1 I can't, but 4 from 4 (which is what I do by at *qrs.*) there remains 0, but the top 1 is 1, which I set down. Again, I carry 1 to 17 is 18 from 11 I can't, but 18 from 20 there remains 2, and the top 11 makes 13. *Lastly*, I carry 1 to 4 is 5 from 7 there remains 2, and the work is done.

Tyro. I thank you, sir; examples now without words will be sufficient.

Note, If you forget what you do by, turn to your tables in *Addition*.

TROY-WEIGHT.

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.							
Bought	14	-	5	-	15	-	14	Bought	214	-	10	-	17	-	11	
Sold	9	-	9	-	13	-	17	Sold	109	-	10	-	17	-	15	
	<hr/>					<hr/>					<hr/>					
Rem.	4	-	8	-	1	-	21	Remains								

DRY-MEASURE.

<i>Loads bush. pecks pints</i>					<i>Chal. bush. pecks</i>				
<i>Bought</i>	42	-	17	- 2 - 12	<i>Bought</i>	291	-	21 - 2	
<i>Sold</i>	17	-	34	- 2 - 14	<i>Sold</i>	173	-	27 - 3	
<hr/>					<hr/>				
<i>Unfold</i>	24	-	22	- 3 - 14	<i>Unfold</i>				

WINE-MEASURE.

<i>Butts hhd. gall. pints</i>					<i>Hhds. gall. pints</i>				
<i>Bought</i>	14	-	1	- 47 - 0	<i>Bought</i>	64	-	35 - 1	
<i>Sold</i>	9	-	0 - 51 - 5		<i>Sold</i>	17	-	46 - 2	
<hr/>					<hr/>				
<i>Unfold</i>	5	-	0 - 58 - 3		<i>Unfold</i>				

WINCHESTER-MEASURE.

Bar. gall. pints				Butts hhd. bar. gall.			
Brewed	124	- 21	- 4	Brewed	21	- 1	- 1 - 20
Sold out	92	- 27	- 6	Sold out	15	- 1	- 0 - 26
<hr/>				<hr/>			
Unfold	31	- 29	- 6	Unfold			

Note, I have reckoned 36 gallons to the barrel, that being customary in selling beer in most places, as I said before.

CLOTH-MEASURE.

	Yds.	qrs.	N.	Ells	Eng. qrs.	N.	Ells	Fl. qrs.	N.	
Bought	147	- 1	- 0		47	- 1	- 3	47	- 2	- 0
Sold	96	- 3	- 2		18	- 2	- 3	29	- 2	- 1
	<hr/>				<hr/>			<hr/>		
Rem.	50	- 1	- 2							

LONG-MEASURE.

	<i>Deg.</i>	<i>leag.</i>	<i>miles</i>	<i>fur.</i>	<i>rods</i>	<i>yds.</i>	<i>feet</i>	<i>inch.</i>	<i>B. corn</i>
Prom	471	- 14	- 1	- 3	- 57	- 2	- 1	- 10	- 1
Take	167	- 17	- 1	- 5	- 21	- 3	- 1	- 10	- 2
<hr/>									
Rem.	303	- 16	- 2	- 5	- 35	- 3½	- 2	- 11	- 2

LAND-

LAND-MEASURE.

	<i>Acres roods poles yds.</i>	<i>Acres roods poles</i>
From	471 - 2 - 15 - 3	47 - 1 - 15
Take	196 - 2 - 26 - 4	19 - 2 - 15
Rem.	274 - 3 - 28 - 4 $\frac{1}{2}$	

T I M E.

	<i>Yrs. mths. wks. days hours</i>	<i>Days hours min. sec.</i>
From	219 - 10 - 3 - 3 - 17	45 - 17 - 25 - 17
Take	193 - 11 - 3 - 4 - 20	19 - 21 - 43 - 35
Rem.	25 - 11 - 3 - 5 - 21	

Note, In the first example I reckon 13 months to a year.

Tyro. Sir, I am obliged to you for the pains you have taken; I understand you very well, and will try at all those sums you have left undone.

Philo. That is enough, we will finish *Subtraction* then with *Section 3.*

SECTION III.

Containing some useful QUESTIONS to exercise the Learner in both RULES.

Quest. 1. **K**ING Henry the Eighth died 1547; I demand how many years it is, this being now 1783? *Answer* 236 years.

Quest. 2. Suppose this present year 1783, you were 19 years old, what year was you born in? *Ans.* 1764.

Quest. 3. A boy had 1000 marbles, and he lost at 3 different times at play, each 175, and at another time 150; how many has he still in hand? *Ans.* 325.

Rule. Set down 175 three times, add 150 under that, and then add all of them together, and take the sum out of 1000, and the answer will be 325 left.—This rule serves for all that follows.

MO-

M O N E Y.

4. *A* lent *B* £.500, and *B* paid him four times, each £.120 - 10. What is due to *A*? *Ans.* £.18.

5. What sum of money must I add to £.58 - 14 - 6 to make it up £.100? *Ans.* 41 - 5 - 5 $\frac{1}{4}$.

6. *A proof to Quest. 5.* What sum must I take out of £.100, to have the remainder £.58 - 14 - 6 $\frac{1}{4}$. *Answer* £.41 - 5 - 5 $\frac{1}{4}$.

7. A collector of the excise received £.2040 - 14 - 5 $\frac{1}{2}$ and remitted or paid at 3 several times, each £.500 and at another time 100 guineas; what has he still in hand? *Ans.* £.435 - 14 - 5 $\frac{1}{4}$.

A VO I R D U P O I S - W E I G H T.


8. Bought 6 ton, 14 Cwt. of iron, and sold at one time 3 ton, 11 Cwt. 3 qrs. and at several other times sold by retail 15 Cwt. 3 qrs. 17 lb. What remains unsold? *Ans.* 2 ton, 6 Cwt. 1 qr. 11 lb.

T R O Y - W E I G H T.

9. A gentleman delivered to a silversmith 2 lb. 5 oz. 11 dwts. of silver; and he received a silver cup, which weighed 11 oz. 14 dwts. and at another time 6 large spoons, weighing 1 lb. 2 oz. 3 dwts. 14 grs. What weight of silver has the silversmith still in hand? *Ans.* 3 oz. 13 dwts. 10 grs.

L I Q U I D - M E A S U R E.

10. A common brewer has 2 goils of new beer, containing 32 butts, 2 barrels, 24 gallons of strong; and 15 butts, 1 barrel of small. And he sent to one victualler 8 butts, 2 barrels; to another 11 butts; and started into his own storehouses 5 butts, 2 barrels, 27 gallons of the strong; and of the small, 7 butts, 2 barrels; he also sold out by retail, of the small, 4 butts, 23 gallons: I demand what remains of the brewing, both of the goil of strong and the goil of small? *Ans.* 7 butts, 33 gallons of strong, and 3 butts, 1 barrel, 13 gallons of small.

 Note,

Note, I have added the answers, that you may know when you are right or wrong; if you cannot do all these questions as yet, it is of no great signification; you must not stop your progress in going forward, because of that. Now follows

DIALOGUE IV.

SECTION I.

MULTIPLICATION.

Tyro. WHAT is *Multiplication*?

Philo. *Multiplication* is a compendious or short way of *addition*, and teaches us to tell the product of any sum in one line only, which would require several *additions*.

Tyro. What else is required?

Philo. There are three things to be carefully observed in *multiplication*.

First. The *multiplicand*, which stands a-top, and is that number given to be multiplied.

Secondly. The *multiplier*, or sum you multiply by.

Thirdly. The *product* or *answer*, which is the *multiplicand* multiplied by the *multiplier*.

Tyro. Please to explain this a little more.

Philo. Let us take any two numbers, suppose 8 and I set down 8, and place 3 under it; so is 8 the *multiplicand*, 3 is the *multiplier*, and 24 is the *product*, or *answer*, because 3 times 8 make 24. Is this plain enough?

Tyro. Yes, sir, I understand you now.

Philo. Very well, Tyro, then I would have you get the following table by heart, by all means, notwithstanding it is too customarily omitted.

MUL-

Note,

MULTIPLICATION TABLE.

Once 1 is 1					
2 times, or twice	{	2	4	5 times	{
		3	6		5
		4	8		25
		5	10		6
		6	12		30
3 times	{	7	14	6 times	{
		8	16		6
		9	18		36
		7	21		7
		8	24		is
4 times	{	9	27	7 times	{
		4	16		7
		5	20		49
		6	24		8
		7	28		is
5 times	{	8	32	8 times	{
		9	36		8
		5	25		64
		6	30		9
		7	35		is
		8	40		
		9	45		
		6	36		
		7	42		
		8	48		
		9	54		
		7	49		
		8	56		
		9	63		
		8	64		
		9	72		

Table of Twelves

12 times	{	2	24
		3	36
		4	48
		5	60
		6	72
	{	7	84
		8	96
		9	108
		10	120
		11	132
	{	12	144

Note 1. You should be very careful to get the table by heart, as I said before; saying thus, 2 times 2, or twice 2 is 4, twice 3 is 6, twice 4 is 8, twice 5 is 10 &c.

Note 2. You should also at leisure learn to box the table well: that is, to say it backwards, forwards, or any way. Thus you see twice 5 is 10, that is, 5 times 2 is 10. Again, 3 times 9 is 27; so also is 9 times 3 the same. So 6 times 12 is the same as 12 times 6 and 5 times 9 the same as 9 times 5.

Rule. I figure Addition answer.

Ex.

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EXAMPI

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1. To multiply by single Figures.

Rule. Multiply every figure in the *multiplicand* by every figure in the *multiplier*, carrying 1 for every 10, as in *Addition of whole Numbers*, and you have the *product* for answer.

EXAMPLE 1.

Multiplicand 17
Multiplier 5

Product 85

EXAMPLE 2.

Multiplicand 78
Multiplier 6

Product 468

begin with the *multiplier* 5, saying, 5 times 7 is 35, and set down the 5, and I carry 3; then I say 5 times 1 is 5, and 3 that I carried is 8; 10 is the *product* for answer 85. Again, in *Ex. 2*, I say, 6 times 8 is 48; that is 8 and I carry 4; then 6 times 7 is 42, and 4 I carried is 46: so this *product* 468.

To prove the Work by Addition.

I set the *multiplicand* 17 down 5 times; and the *multiplicand* 78 I set down 6 times, and adding them together, I find 85, and 468, as before; which you may do on your slate at leisure.

Tyro. I see plainly that *multiplication* saves the trouble of many *additions*.

Philo. There is another way to prove *addition*; but it is of no manner of signification, we will lay aside these puzzling curiosities, and proceed to

EXAMPLE 3.

Multiply 472
by 8

Answer 3776

EXAMPLE 4.

24970
by 9

Ans. 224730

EXAMPLE 5.

and 590407
by 7

Ans. 4132849

Tyro.

Tyro. I say, 8 times 2 (or twice 8) is 16, 6 and carry 1; then 8 times 7 is 56, and 1 is 57, that is 57 and I carry 5; then 8 times 4 is 32, and 5 I carried is 37. In *Ex. 4*, I say, 9 times (0) nothing is 0; then 9 times 7 is 63, that is, 3 and I carry 6; then 9 times 6 is 54, and 6 I carried is 60, that is 0 and I carry 6; then 9 times 4 is 36, and 6 is 42, that is 2 and I carry 4; and *lastly*, 9 times 2 is 18, and 4 I carried is 22. Also in *Ex. 5*, I say, 7 times 7 is 49, that is 9 and carry 4; 7 times 0 is 0, but 4 is 4 still; then 7 times 6 is 42, that is 2 and I carry 4; then 7 times 5 is 35, and 4 is 39, that is 9 and I carry 3; then 7 times 3 is 21, that is 1 and I carry 2; then 7 times 2 is 14, that is 4 and I carry 1; then 7 times 1 is 7, and 1 is 8. And *lastly*, 7 times 0 is 0, but 1 is 1. And the answer is 41.

Philo. Very well done indeed! I leave these two examples for practice.

$$\begin{array}{r} \text{Multiply } 49997296 \\ \text{by } 9 \\ \hline \end{array}$$

Answer

$$\begin{array}{r} \text{and } 94171470 \\ \text{by } 8 \\ \hline \end{array}$$

Answer

Tyro. Please to give me an example or two to multiply by 12 in one line; for I know it is done much quicker and it is as easy as to make two of it.

EXAMPLES with twelves.

$$\begin{array}{r} \text{Multiply } 42576 \\ \text{by } 12 \\ \hline \end{array}$$

Answer 510912

$$\begin{array}{r} \text{and } 994079 \\ \text{by } 12 \\ \hline \end{array}$$

Anf. 11928948

$$\begin{array}{r} \text{and } 99807 \\ \text{by } 12 \\ \hline \end{array}$$

Anf.

First, 12 times 6 is 72, that is, 2 and I carry 7; then 12 times 7 is 84, and 7 I carried is 91, that is 1 and carry 9; then 12 times 5 is 60, and 9 I carried is 69, that is 9 and I carry 6; then 12 times 2 is 24, and 6 is 30, that is 0 and I carry 3; and *lastly*, 12 times 4 is 48, and 3 is 51. And thus, by having the *table of twelves* perfectly by heart, every sum will be easy to you.

Tyro. It is easy enough, I see: please now to show me how you multiply by 2, 3, 4, or more figures.

Philo. This you will soon understand:

2. Of multiplying by several Figures.

Rule. When there are several figures in the *multiplier*, you begin at the first figure in the units place, and multiply it through; or into every figure of the *multiplend*, as you have done before. This being done, multiply every figure in the *multiplend* by the second figure of the *multiplier*; only observe to set the first figure of the second row under the *tens place* of the first figure; and thus you go on with the third figure in the *multiplier*, placing the *units place* of each succeeding row under the *tens place* of the row that is above it, when you have gone through every figure of the *multiplier*. Then draw a line under all the rows of figures, and add them together, and the sum is the true *product* or *answer*.

Note. Be sure you remember that you set your figures right under one another, or else in a large sum you will be puzzled to add the work together.

EXAMPLES with several Figures.

EXAMPLE 1.

Multiply 89
by 47

623
356
—

Answer 4183

PROOF.

Multiply 47
by 89

423
376
—

The same 4183

In *Example 1*, I say, 7 times 9 is 63, that is 3 and I carry 6; then 7 times 8 is 56, and 6 I carried is 62, which I set down: so is the first line or row finished. Then I take the second figure of the *multiplier*, saying, 4 times 9 is 36, which 6 I set under the 2, or *tens place* of the first line, and carry 3; and then I say, 4 times 8 is 32, and 3 that I carried is 35, placing the 5 under the 6, and the 3 quite out towards the left hand. Then I add these up in order, as they stand, saying 3 + 6 = 9; then 9 and 2 is 11; again, 11 and 6 is 17, that is 7 and I carry 1 to the 3 is 4.

To

To prove MULTIPLICATION.

It is a common way to prove *multiplication* by the cross; but it is subject to so many errors, that, in the first place, it is no proof at all to a learner, but rather a corruption. The best way therefore is this: take the *multiplicand*, and set it below, and the *multiplier* a-top; then, change the *multiplicand* into the *multiplier*, and proceed as before directed, and if the *product* be the same as before, your work is entirely right. See the next examples wrought at large both ways.

EXAMPLE 2.

Multiplicand 895*Multiplier* 798

7160

8055

6265

Product 714210

PROOF.

Multiplicand 798*Multiplier* 895

3990

7182

6384

The same 714210

First, I say, 8 times 5 is just 40; therefore I set down the 0, and I carry 4; then 8 times 9 is 72, and 4 I carried is 76, that is 6 and I carry 7; then 8 times 8 is 64, and 7 I carried is 71. Now I take the second figure saying, 9 times 5 is 45; which 5 I place under the second figure, or *tens place* of the first line, and carry 4 saying, 9 times 9 is 81, and 4 is 85, which 5 I place under the figure 1, and carry 8 to the next figure, saying, 9 times 8 is 72, and 8 that I carried is 80. And now I come to the last figure of the *multiplier*, saying, 7 times 5 is 35, which 5 I place under the second figure of the last line, and carry 3 to the next figure, saying, 7 times 9 is 63, and 3 I carried is 66, that is 6 and I carry 6; then 7 times 8 is 56, and 6 is 62. *Lastly*, I add these in order as they stand, and find the *product* 714210.

The proof of this example is worthy your observation, *Tyro*; for be the sum ever so large, if you change the *multiplicand* into the *multiplier's* place, and multiply right, you will find the *product* always the same.

Of MULTIPLICATION.

69

Tyro. I see it plainly, sir; and I could not have thought multiplication had been so easy.

Philo. Nothing easier, when the *table* is once well learned. I shall now give you two more examples, and leave the rest undone for practice.

EXAMPLE 3.

$$\begin{array}{r}
 \text{Multiply} \quad 913876 \\
 \text{by} \quad 8759 \\
 \hline
 8224884 \\
 4569380 \\
 6397132 \\
 7311008 \\
 \hline
 8004639884
 \end{array}$$

EXAMPLE 4.

$$\begin{array}{r}
 \text{and} \quad 749076 \\
 \text{by} \quad 1795 \\
 \hline
 3745380 \\
 6741684 \\
 5243532 \\
 749076 \\
 \hline
 1344591420
 \end{array}$$

EXAMPLES for Practice.

$$\begin{array}{r}
 \text{Multiply} \quad 4567879 \\
 \text{by} \quad 45769 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Multiply} \quad 9567950 \\
 \text{by} \quad 32796 \\
 \hline
 \end{array}$$

Note. Remember, *Tyro*, that when you multiply several figures, that you always set the first figure of the second line under the second figure of the first line, and the first figure of the third line under the second figure of the second line: and thus go on, leaving the figure out towards the right-hand every line, setting the next line one figure more towards the left-hand, and every figure under one another in their proper place.

Tyro. I understand you well, sir.

Philo. Then I will set you but one example more, which you may prove yourself.

Multiply

Of MULTIPLICATION.

$$\begin{array}{r}
 \text{Multiply} \quad 987654321 \\
 \text{by} \quad 123456789 \\
 \hline
 8888888889 \\
 7901234568 \\
 6913580247 \\
 5925925926 \\
 4938271605 \\
 3950617284 \\
 2962962963 \\
 1975308642 \\
 987654321 \\
 \hline
 \end{array}$$

Answer 121932631112635269

P R O O F.

$$\begin{array}{r}
 \text{Multiply} \quad 123456789 \\
 \text{by} \quad 987654321 \\
 \hline
 \end{array}$$

3. Of Cyphers between the Figures, &c.

The most difficult thing in this rule is, when the *multiplier* has some figures, some cyphers; but if you be careful to mind the work of an example or two, you will soon understand it, as I shall explain them in words.

EXAMPLE 1.

$$\begin{array}{r}
 \text{Multiply} \quad 49657 \\
 \text{by} \quad 30705 \\
 \hline
 248285 \\
 3475990 \\
 1489710 \\
 \hline
 \end{array}$$

Answer 1524718185

EXAMPLE 2.

$$\begin{array}{r}
 \text{Multiply} \quad 7564965 \\
 \text{by} \quad 500300700 \\
 \hline
 5295475500 \\
 2269489500 \\
 3782482500 \\
 \hline
 \end{array}$$

Answer 3784757284975500

Note, Pray observe, that you read the following instructions once or twice over, if you do not understand the work.

First

First, I begin at the 5, and multiply all the top figures by it, as before; now, as the next figure of the *multiplier* is a cypher, it is of no signification to multiply by *that*, because it will produce nothing but a line of cyphers: therefore I bring it down, that is, I set a cypher under the figure 8, and then begin to multiply by the 7, saying, 7 times 7 is 49, 9 and I carry 4, which 9 I set down in the same line, on the left-hand of the cypher, and go on still to multiply by the 7, then as the next figure is a cypher again, I set down a cypher under it, as I did before, leaving the 9, and the cypher which stands by it, in the last line, both standing out towards the right-hand. This being done, I only multiply by the 3, and set it in the same line, and add all up, and the work is done.

Again, in *Example 2*, there are three cyphers at the beginning of the *multiplier*, therefore I set three cyphers under the line, right under them; then multiply by the 7, as in all other sums; saying, 7 times 5 is 35, &c. Then I come to two cyphers more in the *multiplier*, and therefore set two cyphers right under them, and then multiply by the 3, and place the product by the side of the two cyphers; then, as there are two cyphers, I again set down two cyphers, thus 00, under them, and then multiply by the 5, placing the product by the side of them. *Lastly*, I add the sum up, and it stands, and it is done.

Note, These examples are proved like the others, by setting the *multiplier* a-top, and the *multiplicand* under

More Examples for Practice.

<p>Multiply by</p>	<p>570965 2400500</p> <hr style="width: 100%;"/>	<p>and</p>	<p>71900072 5000960000</p> <hr style="width: 100%;"/>
------------------------	--	------------	---

And now, *Tyro*, I shall finish this section with shewing you something of *contractions*, or *compendiums*.

4. Of CONTRACTIONS.

When any number of figures are given to be multiplied by 10, 20, 30, 40, 80, 90, 120, 1200; then set the cyphers out, and multiply by the figure or figures, and place the *product* by the side of the cyphers, as before gives the *answer*.

$$\begin{array}{r} \text{Multiply} \quad 275 \\ \text{by} \quad 20 \\ \hline \end{array}$$

$$\text{Answer} \quad 5500$$

$$\begin{array}{r} \text{and} \quad 3275 \\ \text{by} \quad 90 \\ \hline \end{array}$$

$$\text{Ans. } 294750$$

$$\begin{array}{r} \text{and} \quad 675 \\ \text{by} \quad 12 \\ \hline \end{array}$$

$$\text{Ans. } 8100$$

2. When you are to multiply by 1, the *answer* will be the same as the *multiplicand* itself is. If you are to multiply by 10, add 1 cypher to the *multiplicand*; if by 100, add two cyphers; if by 1000, add three cyphers. See the work two ways.

$$\begin{array}{r} \text{Multiply} \quad 1753 \\ \text{by} \quad 100 \\ \hline \end{array}$$

$$\text{Answer} \quad 175300$$

$$\begin{array}{r} 1753 \\ 1000 \\ \hline \end{array}$$

$$\text{Ans. } 1753000$$

$$\begin{array}{r} 1753 \\ 100 \\ \hline \end{array}$$

$$\text{Ans. } 175300$$

Or rather thus:

Multiply 1753 by 1	answer the same, viz.	1753
by 10	add 1 cypher,	ans. 17530
by 100	add 2 cyphers,	ans. 175300
by 1000	add 3 cyphers,	ans. 1753000
by 10000	add 4 cyphers,	ans. 17530000

And thus for as many cyphers as you please.

Tyro. The examples are quite plain and easy.

Phil. I am glad that you understand them; for I shall shew you *that* which will serve you for all common business, in casting up any sort of goods; and pray be careful to mind it well; for there is not a more useful thing in all common arithmetic, it being a *the* way of working the *Rule of Three* without *Division*.

Questi

Questions performed by Multiplication.

Tyro. How are these questions performed?

Philo. By the following rule, viz.

Multiply the price by the given number, or quantity, and carry 1 for every 4, 12, and 20, as you do in *Addition of Money*, and 1 for every 10 in the pounds, as in *Whole numbers*.

An example or two well explained will soon make easy to you.

$$\begin{array}{r}
 \text{Quest. 1. What cost 3 ells, at } \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 0 & - & 5 & - & 3 \end{array} \text{ an ell?} \\
 \hline
 \text{£.0} & - & 15 & - & 9
 \end{array}$$

Here I multiply the price by the quantity, saying, 3 times 3 is 9 pence, which I place under the pence; and then I say, 3 times 5 shillings is 15, which I place under the shillings, and the answer is 15 shillings and 9 pence.

You must remember, that 3 ells, at 5s. 3d. an ell, is the same as 3 yards, or 3 grofs, or 3 gallons, or any other name whatsoever; for it is only 3 times 5 shillings and 3 pence.

$$\begin{array}{r}
 \text{2. What come 5 grofs of } \left. \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 0 & - & 6 & - & 8 \end{array} \right\} \text{ a grofs?} \\
 \hline
 \text{£.1} & - & 13 & - & 4
 \end{array}$$

Here I multiply the price by 5, saying, 5 times 8 is 40 pence, (which is 3 shillings and 4 pence) therefore I set down 4 under the pence, and carry 3, and then 5 times 6 shillings is 30, and 3 I carried is 33 shillings, which is 1l. 13s. that is 13 and I carry 1; and then I say, 5 times 0 is 0, but 1 is 1.

3. How much does 7 times 8 shillings and 9 pence amount to? Or,

What cost 7 reams of paper, at $\left. \begin{array}{l} \text{£.} \quad \text{s.} \quad \text{d.} \\ 0 - 8 - 9 \end{array} \right\}$ a ream?

7

£. 3 - 1 - 3 *Ans.*

Here I say, 7 times 9 is 63 pence, which is 5 shilling and 3 pence, that is 3 and I carry 5; then 7 times 8 is 56 shillings, and 5 I carried is 61, that is 3*l.* 1*s.* which is the same, it is 3 twenties, and 1 over; therefore I set 1 under the shillings, and carry 3 to the pounds, saying, 7 times 0 is 0, but 3 is 3; so is the answer 3*l.* 1*s.* 3*d.*

Note, Though I have set cyphers in the place of pounds in these examples, and have also set £. s. d. over the sums, yet it is better to leave out the cyphers and more practicable to make only a great £. before the pounds, as in the following examples.

4. What cost 9 gallons of brandy, or any thing else at ----- £. --- 8 - 3 $\frac{1}{4}$ a gallon?

9

£. 3 - 14 - 5 $\frac{1}{4}$

Here I multiply the price by 9, beginning at the farthings, saying, 9 farthings is 2 pence farthing, that is 1 farthing, and I carry 2 pence; then 9 times 3 is 27 pence, and 2 pence I carried is 29 pence, which is 2 shillings and 5 pence; therefore I set down 5 under the pence, and carry 2, saying, 9 times 8 is 72 shillings, and 2 I carried is 74. Now 74 shillings is 3*l.* 14*s.* therefore, as there is nothing in the pounds to multiply, I set down 3*l.* 14*s.* viz. the 14 under the shillings place and 3 towards the left hand.

Tyro. This is very pretty, and very easy: please try me with an example.

Philo. There is no doubt but you will do it.

5. A Gentleman gave ten poor Widows three crowns a piece: how much did he give in all? or the What cost 10 bushels of any thing, at £. --- 7 - 4 a bushel?

£. 3 - 15

Of MULTIPLICATION.

75

Tyro. I set down 3 half crowns, that is 7s. 6d. and multiply it by 10, saying, 10 times 6 is 60 pence, that is 5 shillings; therefore I set down nothing (0) and carry 5: then I say, 10 times 7 is 70, and 5 I carried is 75 shillings, which is 3l. 15s. So is the answer, 3 - 15.

6. What comes 12 months } £. 1 - 8 - 6 $\frac{1}{4}$ a month?
ages to, at 12

how much is 12 times *Ans.* 17 - 2 - 3
£. 1 - 8 - 6 $\frac{1}{4}$?

Here I say, 12 farthings is 3 pence, that is 0 and I carry 3; then 12 times 6 is 72, and 3 I carried is 75 pence, which is 6s. 3d. I therefore set down 3, and carry 6; then I say, 12 times 8 is 96 shillings, and 6 I carried is 102 shillings, which is 5l. 2s. set down 2, and carry 5; then I say, 12 times 1 is 12, and 5 I carried is 17. So is the answer £. 17 - 2 - 3.

2. Of double Figures in the Shillings.

When you come to double figures, such as 14, 17, 18, the like; multiply them like *whole numbers*, and then cut out the twenties, that is, count how many times 20 can have in the number, and it is done. There are several other ways I shall shew you, and always take *that* which appears easiest to you, till perfect.

What cost five sheep, at £. 1 - 17 - 6 each?

What is 5 times £. 1 - 17 - 6

5

Ans. £. 9 - 7 - 6

Here I say, 5 times 6 is 30 pence, or 2s. 6d. that is 6 I carry 2; then I multiply the whole 17 by 5, saying, 5 times 7 is 35, and 2 I carried is 37, 7 and I go 3; 5 times 1 is 5, and 3 is 8, that is 87 shillings, which is 4l. 7s. that is 7 and I carry 4; then 5 times 1 is 5, and 4 is 9.

Or thus:

I multiply first the 7 by the 5, setting down the product on a slate or piece of paper, saying, 5 times 7 is 35 and 2 I carried is 37 shillings, or 1*l.* 17*s.* then I say 5 times 10 is 50 shillings, or 2*l.* 10*s.* and add this to 17*s.* it makes 4*l.* 7*s.* as before. See the next example.

8. What cost 7 gallons, at £. 1 - 15 - 9 $\frac{1}{4}$ a gallon

$$\begin{array}{r} 7 \\ \hline \text{£. } 12 - 10 - 4\frac{3}{4} \end{array}$$

Here I say, 7 farthings is 1 penny 3 farthings, therefore I set down $\frac{3}{4}$, and carry 1; then 7 times 9 is 63 and 1 is 64 pence, that is 5*s.* and 4*d.* 4 and I carry 3; then 7 times 5 is 35, and 5 is 40, 0 and I carry 4; 7 times 1 is 7, and 4 is 11, that is 110 shillings in all, which is 5*l.* 10*s.* that is 10 and I carry 5; and lastly, 7 times 1 is 7, and 5 is 12.

Or thus, 7 times 5 is 35, and 5 is 40 shillings, that is 2*l.* which I set down any where on a slate; then I say, 7 times 10 shillings is 3*l.* 10*s.* Now 3*l.* 10*s.* and 2*l.* is 5*l.* 10*s.* as before. Again,

9. What cost 12 quarters of malt, at £. 1 - 14 - a quarter?

$$\text{Ans. } 20 - 14 -$$

Here I say, 12 6 pences is 6 shillings, that is 0 and I carry 6; then 12 times 4 is 48, and 6 is 54, 4 and I carry 5; 12 times 1 is 12, and 5 is 17; that is 17 shillings, which is 8*l.* 14*s.* that is 14 and I carry 8; then 12 times 1 is 12, and 8 is 20 pounds. Or,

By the second way, 12 6 pences is 6 shillings, as before 0 and I carry 6; then 12 times 4 is 48, and 6 is 54, that is 2*l.* 14*s.* then 12 times 10 shillings is 6*l.* Now 6*l.* and 2*l.* 14*s.* is 8*l.* 14*s.* as before, that is 14 and I carry 8; then 12 times 1 is 12, and 8 is 20.

Tyro. I understand both the ways very well.

Philo. Take which you think is easiest, they both answer the same end.

Tyro. But suppose I have a larger quantity than how must I multiply the price by that number?

Philo. By the following rule.

3. *When there are two Figures to multiply by.*

Rule. When the given quantity is more than 12, and contains any such numbers as can be found in the *Multiplication-table*, then find any two numbers, which when multiplied together will make the given number, and then multiply the given price by any one of those figures, and that product multiply by the other number, and that product is the answer. Thus, suppose I am to multiply by 15, I find 3 times 5 is 15; therefore I multiply by the 3 first, and set down the product; then I multiply this product by 5, and that gives me the answer for 15, because 3 times 5, or 5 times 3 make that number. Again, suppose I were to multiply by 24, I find 4 times 6 is 24; therefore I first multiply by 4, and then multiply that product by 6, which is the answer: or otherwise, as 3 times 8 is 24, I may therefore multiply first by 3, and then by 8. So if the number be 63, I find 7 times 9, or 9 times 7 is 63; therefore, I multiply the price by any one of these figures, and that product multiply by the other for the true answer.

What cost 15 ells of holland, at £. 7 - 6 an ell?
Here 3 times 5 is 15, multiply first by 3

The price of 3 ells	—	£. 1 - 2 - 6
Multiply by 5		5
Price of 15		£. 5 - 12 - 6 <i>Ans.</i>

Work at Length.

First I say, 3 six-pences is 1s. 6d. that is 6 and I carry 1; then 3 times 7 is 21, and 1 I carried is 22 pences, which is 1l. 2s. So is the price of 3 ells, 1s. 6d. Now I multiply this 1l. 2s. 6d. by 5; (because 5 times 3 is 15) saying, 5 times 6 is 30 pence, or 2s. 6d. or 5 six-pences is 2s. 6d. that is 6 and I carry 2; then 5 times 2 is 10, and 2 is 12, which I set down. Lastly, I say, 5 times 1 is 5; so the answer for 15 ells, at 7s. 6d. an ell, is £. 5 - 12 - 6

11. What cost 24 chaldrons }
 of coals, at } £. 1 - 11 - 6 a chaldron
 4 times 6 is 24, first multiply by 4

The price of 4 chald. $\begin{array}{r} 6 - 6 - 0 \\ \hline 6 \end{array}$

The price of 24 chald. £. 37 - 16 - 0 *Ans.*

Proved another way £. 1 - 11 - 6
 3 times 8 is 24, multiply by 3

Price of 3 chald. $\begin{array}{r} 4 - 14 - 6 \\ \hline \text{Multiply by } 8 \end{array}$

Price of 24 chald. £. 37 - 16 - 0 as above

12. What cost 35 sheep, at £. 1 - 15 - 7 each?
 5 times 7 is 35, multiply first by 5

Price of 5 $\begin{array}{r} 8 - 17 - 11 \\ \hline 7 \end{array}$

Price of 35 £. 62 - 5 - 5 *Ans.*

13. What cost 48 load, at £. 3 - 10 - 6 a load?
 6 times 8 is 48, multiply by 6

Price of 6 $\begin{array}{r} £. 21 - 3 - 0 \\ \hline \text{Multiply by } 8 \end{array}$

Price of 48 £. 169 - 4 - 0

14. What cost 72 pieces of }
 Holland, at } £. 2 - 3 - 5 a piece
 6 times 12, or 12 times 6 is 72, by 12

Price of 12 $\begin{array}{r} 26 - 1 - 0 \\ \hline \text{Multiply by } 6 \end{array}$

Price of 72 £. 156 - 6 - 0 *Ans.*

15. What cost 108 quar-tern loaves, at } £. 0 - 0 - 5 $\frac{3}{4}$ a piece?
12 times 9 is 108, multiply by 12

$$\begin{array}{r} \text{Price of } 12 \quad 0 - 5 - 9 \\ \text{Multiply by } 9 \quad 9 \\ \hline \text{Price of } 108 \quad \text{£. } 2 - 11 - 9 \end{array}$$

Note 1. I have here indulged you with cyphers, because I would do every thing that is easy to your understanding; but really, *Tyro*, it is better without them. Set down now 5 pence 3 farthings any where.

$$\begin{array}{r} 5 \frac{3}{4} \\ 12 \\ \hline - 5 - 9 \\ 9 \\ \hline \text{£. } 2 - 11 - 9 \text{ Answer.} \end{array}$$

Note 2. This is the only way, if you have a mind to do things quick, and off hand; for tho' there be a form in some schools to the contrary, and you may hitherto have been instructed in that way; yet, as I observed before, when you come to do a sum, you are not bound to set it down in the very form you were taught at school; for while you are doing *that*, another will answer the question over and over.

Tyro. I understand you, sir, very well, and thank you for your care; but it is only use, as you observed; for I think one full as easy as the other, and I grant it is much better for the quick dispatching of business.

16. What comes the cloath-
ing of 144 soldiers to, at } £. 3 - 12 each man?
12 times 12 is 144, multiply by 12

$$\begin{array}{r} 12 \text{ cost} \quad 43 - 4 \\ 12 \text{ again by } 12 \\ \hline 144 \text{ cost} \quad \text{£. } 518 - 8 \text{ Ans.} \end{array}$$

E 4

Tyro.

Tyro. You need not give me any more examples, but pray, suppose the quantity, or given number, be such as does not fall under, or cannot be found in the *Multiplication table*, such as 17, 26, 38, 59, or the like; how then?

Philo. This is as easy as the other, as appears by the following rule.

4. - *Of Numbers that are not to be found in the Multiplication Table, and how to work by them.*

Rule. When the given number does not fall under the common course of the table, such as 17, 38, 59, or the like; then take any two figures that will come the nearest to the number (but not exceeding it) and work with them as in the former examples, and then add the price to that product as many times as there are odd numbers, and the work is done.

We will give you examples of 17, 38, and 59.

17. What cost 17 grofs of corks, at £. 10 - 6½ a grofs.
4 times 4 is 16, and 1 is 17. Mult. by 4

Price of 4 is	2 - 2 - 2
4	4
<hr/>	

Price of 16 is	8 - 8 - 8
Add the price of 1 grofs, viz.	10 - 6½
<hr/>	

17 is £. 8 - 19 - 2½ *Ans.*

18. What cost 38 loads of hay, at £. 1 - 11 - 6 a load.
6 times 6 is 36, and 2 is 38. Mult. by 6

Price of 6 is	9 - 9 -
6	6
<hr/>	

Price of 36 is	56 - 14 -
Mult. the price 1l. 11s. 6d. by 2 gives	3 - 3 - add
<hr/>	

Price of 38 is £. 59 - 17 - *Ans.*

19. What

9. What is the value of 59 ports, at £. 1 - 16 - each?
7 times 8 is 56, and 3 is 59. Mult. by 7

Value of 7 is $\begin{array}{r} 12 - 12 - \\ 8 \text{ by } 8 \end{array}$

Value of 56 is $\begin{array}{r} 100 - 16 - \end{array}$

Then multiply 1*l.* 16*s.* by 3 is $\begin{array}{r} 5 - 8 - \text{add} \end{array}$

Value of 59 £. 106 - 4 -

This may be done another Way.

As you said before, 7 times 8 is 56, and 3 added make 59. So now you may say, 6 times 10 is 60; and then take or subtract the value of one piece out of that product, and the answer will be the same as 59.

1 port is £. 1 - 16 -
6 times 6 is 60, first by 6

Value of 6 is $\begin{array}{r} 10 - 16 \\ 10 \end{array}$

Value of 60 is $\begin{array}{r} 108 - \end{array}$

Subtract 1 piece, viz. $\begin{array}{r} 1 - 16 - \text{subtract} \end{array}$

59 *Ans.* 106 - 4 - as above

Note 1. As some boys learn sooner by one method than another, some masters find it easier to teach them to calculate by even tens, and add the odd numbers after. Thus, if it were 43, then multiply by 10, then 4, gives 40, and then add the 3 odd numbers. So is $10 \times 6 + 5 = 65$, and thus for any numbers.

Note 2. If you are perfect in what has already been shown you, it will be very easy to perform any sum in common arithmetic, relating to business, not only within the compass of the table, but in much larger numbers. As for instance; suppose it was required to find the value of 1000 moidores; or what it will cost to pay 1000 men, at 1*l.* 7*s.* a man? Here 10 times 10 make

E 5.

make 100, and 10 times 100 make 1000. Multiply therefore *1l. 7s.* by 10; then that product by 10; and this last product by 10 will give the answer, *viz. £. 1350.*

QUESTIONS *undone for practice.*

1. What comes 33 dozen of candles to, at *4s. 9d.* a dozen? *Answer £. 7 - 16 - 9.* Multiply by 3, and then by 11.

2. How much is 46 pistoles, at *17s. 6d.* each? or what cost 46 pieces of *Irish*, at *17s. 6d.* a piece? *Answer £. 40 - 5.* Multiply by 5, then by 9, and add *17s. 6d.* more for the odd 1.

3. What cost 63 barrels, at *£. 1 - 5 - 9* a barrel? *Ans. £. 81 - 2 - 3.*

4. What comes 49 months salary to, at 3 guineas a month? *Ans. £. 154 - 7.*

5. What comes 105 gallons of rum to, at *7s. 6d.* a gallon? *Ans. 39 - 11 - 10½.* 10 times 10 make 100 and then multiply *7s. 6d. ½* by 5, the odd 5 gallons, and add it to the last product.

6. If I spend *1d. ¾*, or 7 farthings a day, how much is that a year, allowing 365 days? *Ans. £. 2 - 13 - 2½.*

First, 10 times 10 makes 100; then multiply the answer of 100 by 3, it gives you the expences of 300 days *viz. 2l. 3s. 9d.* Now there are but 65 days behind therefore set down 7 farthings in another place, and multiply it by 8, and that product multiplied by 8 again makes 64, and then add once *1d. ¾* for the odd day, gives *9s. 3d. ¾* for the 65 days, which added to the 300 days gives *£. 2 - 13 - 2¾.* *Secondly*, or thus, which is shorter 11 times 11 is 121; then multiply the product of 11 by 3, gives 363, and then add 2 times *1d. ¾*, *viz. 3d.* to that product, gives *£. 2 - 13 - 2¾*, as before.

As I have added the answers to these questions, hope, *Tyro*, you will try to prove them at large: for you mind and make yourself master of this rule, you will be able to cast up any common thing as quick, quicker than by any other method, and with far greater ease.

As for *cross multiplication*, *viz.* multiplying money by money, or feet and inches by feet and inches, it

better let alone for the present; therefore we will proceed to *Division*.

DIALOGUE V.

SECTION I.

Of DIVISION.

Tyro. **W**HAT is *Division*, and what does it teach?

Philo. *Division* is just the reverse of *multiplication*; for, as any sum is encreased as many times as the figure you multiply by; so in *division*, the number is decreased, or divided into as many parts as the value of the figures you divide by.

Tyro. What is to be learned or observed in this rule?

Philo. There are four things very necessary to be known in *division*, viz.

First, The *Dividend*, which is the sum given to be divided.

Secondly, The *Divisor*, or the number you are to divide by.

Thirdly, The *Quotient*, or *Answer*, which shews you how many times the *Divisor* is contained in the *Dividend*; or into how many parts the *Dividend* is divided.

Fourthly, The *Remainder*, which is a fractional part of the *Quotient*; but this does not concern you as yet.

Tyro. Am I to get these by heart?

Philo. You are not bound to get them just word for word; but you should understand well what the first 3 mean.

Tyro. Be so kind, sir, as to explain them a little more for me.

Philo. Observe then: let us take any two numbers, suppose 24, and 6; now 24 is the *Dividend*, and 6 is the *Divisor*. Then I ask how many times 6 I can have in 24, and the answer is 4; which 4 is called the *Quotient*. So also, suppose it was required to divide 108 by 12 into 12 parts, then every part will be 9; for 12 times

E. 6

times 9 is 108. Now 108 is the *Dividend* or sum to be divided; 12 is the *Divisor*, or what you divide by; and 9 is the *Quotient*.

Tyro. Now, sir, I apprehend you quite well.

Philo. Then I will proceed to some examples in single figures; but pray observe the following rule.

1. *Of dividing by single Figures in one line only.*

Rule. First ask how many times the *Divisor* is contained in the first figure of the *Dividend*, and if the *Divisor* be larger than the first figure in the *Dividend*, then seek how many times you can have it in the first two figures in the *Dividend*, and set the figure down accordingly; and if any thing remains from the first figure in the *Dividend*, carry it to the second; and if any thing remains in the second figure, carry it to the third; remembering always in this short *Division*, that if one remains you call it 10, if two remains you call it 20, if 5, 50, and so on, carrying the remainder of one figure to another in your mind.

Note. To make the work both shorter and easier, remember that 2's is read *two's*, 3's *three's*, 5's is *five's*, 12's is *twelve's*, and so for any other figure: thus, the 7's in 14 is read thus, the *sevens* in 14; and the 6's in 24, is the *sixes* in 24; which is the same as if I should ask, how many *sixes* can I have in 24, but only shorter and more convenient.

An example or two will, with care, make it familiar to you.

Ex. 1. *Dividend.*

Divisor 3)39

Quotient 13 *Ans.*

3

Proof 39

Ex. 2. *Dividend.*

Divisor 4)168

Quotient 42 *Ans.*

4

Proof 168

Now observe: In *Ex. 1*, I ask how many *three's* I can have in 3; or I say the 3's in 3 is once; therefore I set down 1 under the 3 in the *dividend*, and as there re-

mains

remains nothing over, I ask how many 3's I can have in 9; or say, the 3's in 9 is 3 times 3, and nothing over; therefore I set down 3 under the 9, and it is done. Again,

In *Ex. 2*, I say, the 4's in 1 I can't have; but taking the next figure to it, *viz.* 6, I say the 4's in 16 is just 4 times, therefore I set down 4 under the 6 in the *dividend*, and as there is nothing remains, I only say, the 4's in 8 is twice, therefore I set down 2 under the 8, and it is done. Now to prove it I multiply 42, the *quotient*, by 4, the *divisor*, and find it 168, like the *dividend*.

Ex. 3. Dividend.

Divisor 8)37192

Answer 4649
8

Proof 37192

Ex. 4. Dividend.

Divisor 9)245052

Answer 27228
9

Proof 245052

Now observe, *Tyro*, in *Ex. 3*, I divide by 8, saying, the 8's in 3 I can't, but the 8's in 37 is 4 times 8 is 32, and 5 over; I therefore set down 4 under the 7, and carry 5 to the next figure, which is 1, which I now call 31; (for what you carry from one figure you must always place before the next figure) then I say, the 8's in 31 is 6 times 8 is 48, and 3 over; which 3 I now carry to the 9, and it is 39; therefore I say, the 8's in 39 is 4 times 8 is 32, and 7 over; this 7 I now place before the 2, and it is 72; then I say, the 8's in 72 is just 9 times, and the work is done. To prove it, I multiply the *quotient* or answer by the *divisor* 8, and find the *product* the same as the *dividend*. In *Ex. 4*, I say, the 9's in 2 I can't, but the 9's in 24 is 2 times, or twice 9 is 18, and 6 over, which 2 I place under the 4, and carry 6 to the next figure, which is 5, and call it 65; then I say, the 9's in 65 is 7 times 9 is 63, and 2 over, which 2 I place before the cypher (0) and it is 20; then I say, the 9's in 20 is twice 9 is 18, and 2 over, which I carry to the 5, and it is 25; then I say, the 9's in 25 is twice 9 is 18, and 7 over, which I set before the last figure 2, and it is 72; then I say, the 9's in 72 is just 8 times.

times. To prove it, I multiply the *answer* by 9, and the *product* will be the same as the *dividend*.

EXAMPLE 5.
Divide by 7)417296

Answer 59613 - 5*
7

Proof 417296

* Here, in *Example 5*, there is 5 remains at last; therefore I set it at the end of the answer, parting it with a stroke thus, -5; and when I prove the work, I multiply by 7, and take the remaining 5 in, saying, 7 times 3 is 21, and 5 is 26, 6 and I carry 2, &c.

EXAMPLE 7.
8)716206

Answer 89525-6
8

Proof 716206

EXAMPLE 6.
Divide by 9)9471636

Answer 1052404
9

Proof 9471636

EXAMPLE 1.
5)12631908

Answer 2526381-3
5

Proof 12631908

More Examples for Practice.

7)697220716

9)407217943

Tyro. I understand what you have shewn me very well; but pray, before you come to long *Division*, give me an example at large to divide by *twelve* the short way.

Philo. I will: let it be required to divide 1479908 by 12, I set it down thus:

EXAMPLE 11.
12)1479908

123325-8

EXAMPLE 12.
12)59904965

4992080-5


Here I divide by 12, saying, the 12's in 14 is one 12, and 2 over, and this 2 I carry to the 7, and it is 27; then I say, the 12's in 27 is twice 12 is 24, and

over

over, that is 39; then I say, the 12's in 39 is 3 times 12 is 36, and 3 over, that is 39 again; therefore I set down another 3, and carry the 3 that is over to the next figure, which is a cypher, calling it 30: then I say, the 12's in 30 is *twice* 12 is 24, and 6 over, which is 68; now the 12's in 68 is 5 times 12 is 60, and 8 over, which I place after the sum, thus, -8, and it is done.

In *Ex. 12*, I say, the 12's in 5 I can't; but the 12's in 59 is 4 times 12 is 48, and 11 over, which 11 carried to, or joined to the next figure 9, is 119; then I say, the 12's in 119 is nine times 12 is 108, and 11 over again, which I now carry to, and join to the next figure 0, and call it 110; then I say, the 12's in 110 is 9 times, and 2 over, which 2 I join to, or carry to the 4, and it is 24; now the 12's in 24 is just *twice*, and nothing over; therefore I now begin a-fresh, at the 9, saying, the 12's in 9 I can't, but the 12's in 96 is just 8 times. *Lastly*, I say, the 12's in 5 I can't have; therefore I set down a cypher under the last figure 5, and drawing a short line, set the 5 that remains after it thus; -5.

Now *to prove* it, I multiply back by 12, and take in the 5, saying, 12 times 0 is 0, but 5 is 5; then 12 times 8 is 96, 6 and I carry 9, &c. till I go thro' the whole.

 *Note*, *Multiplication* is an infallible proof for *Division*; for if you multiply the *quotient* by the *divisor*, you will have the same figures as are in the *dividend*, always remembering to take in the remainder with the first figure you begin to multiply.

Tyro. I humbly thank you, sir, I am quite satisfied with what you have shewn me; and now, if you please, I shall be obliged to you to shew me how to divide by several figures.

Philo. You will soon learn it with care; you must note there are three or four ways to work *Division*; but as my intent is not for curiosity, but improvement, I shall only shew you that method which is most *natural* and *practicable*; you may at any time learn the rest.

2. *Of dividing by 2, 3, or more Figures.*

Rule. First, *seek*, or ask how many times the figures in the *divisor* are contained in the same number of figures in the *dividend*, and put that figure in the *quotient*. Secondly, multiply now the *divisor* by the said figure in the *quotient*, and place it under those figures in the *dividend* that you began to work with, always observing, that the *product* be not larger than the figures in the *dividend*; for if they are, you must rub out, or cancel the figure in the *quotient*, and put one of a less denomination. Thirdly, this being done, *subtract* now the *product* from that part of the *dividend* it stands under, and to the remainder bring down the next figure in the *dividend*, placing or joining it to the last figure of the *remainder*. Then *seek* how many times the *divisor* is contained in these figures; then multiply the *divisor* by the said figure; then *subtract* again; and *lastly*, bring down the next figure in the *dividend*, as before; and thus proceed till you have no more figures in the *dividend* to bring down, and the work is done.

☞ *Note 1.* Every time you *subtract*, observe whether the *remainder* be larger than the *divisor*; for if it be, you must put a larger figure in the *quotient*.

Note 2. Whenever you take a figure down from the *dividend*, and join it to the *remainder*, and that is still less than the *divisor*; then always put a cypher in the *quotient*, and bring down another figure in the *dividend*.

An example or two at large will make it easier.

EXAMPLE I.

<i>Dividend</i>			
<i>Divisor</i> 15	19260	(1284 quotient or answer	
15		<i>Quotient</i> 1284	
<hr/>		<i>Multiply by</i> 15 <i>divisor</i>	
42		<hr/>	
30		6420	
<hr/>		1284	
126		<hr/>	
120		<i>Product</i> 19260 <i>dividend.</i>	
<hr/>			
60			
60			
<hr/>			
0			

Now

Now observe: having two figures in the *divisor*, I ask how many times they are contained in the two first figures of the *dividend*, (*viz.* 19) and I find it *once*; therefore I set 1 in the *quotient*. Secondly, I multiply the *divisor* 15 by 1, saying, *once* 5 is 5, *once* 1 is 1, and place it under the 19. Then, *thirdly*, *subtract* 15 from 19, and there remains 4; and *lastly*, I bring down the next figure in the *dividend*, (*viz.* the 2) and join it by the side of the remainder 4, and it is 42; and now I begin as at the first, and ask how many times 15 are contained in 42, and find it *twice*; therefore I set down 2 in the *quotient*; then I multiply the *divisor* 15 by 2, which is 30, and place it under 42; this done, I *subtract* 30 from 42, and there remains 12; and then again, I bring down the next figure of the *dividend* (*viz.* 6) and place it by the side of 12, and it is 126. Then I ask how many times 15 I can have in 126, and find it 8 times; therefore I put an 8 in the *quotient*, and multiply 15 by it, which is 120, which I place under 126. Then I *subtract* 120 from 126, remains 6, and bring down the last figure in the *dividend*, which is a cipher (0) and it makes 60; then I seek how many times 15 I can have in 60, and find it four times; then multiply 15 by 4, and find it just 60, which I place under the other 60, and the work is done.

R U L E 2.

Note 1. When there are several figures in the *divisor*, it is easier for a learner to ask how many times the first figure of the *divisor* is contained in the first figure of the *dividend*, and place the times in the *quotient*; then multiply the whole *divisor* by the *quotient* figure, and if the *product* be more than the figures which belong to the *dividend*, you must make trial of a less figure, and put it in the *quotient*.

Note 2 If the first figure of the *divisor* be larger than the figure in the *dividend*, then take 2 figures in the *dividend*, and seek how many times the first figure of the *divisor* is contained in them.

Note 3. You must remember, that in making trial how often the first figure in the *divisor* is contained in two figures of the *dividend*, it will sometimes appear to be

be 10 or 12 times ; but observe, it never can be above
at most, and oftentimes not so many as it appears to be

The same example farther demonstrated.

Let it be required to divide 19260 by 15 ?

First, I set the *dividend* down on a slate, and make a couple of crooked lines at the ends of it, in the first of which I place the *divisor*, thus, 15)19260(and the other is to place the *quotient* in.

Secondly, I ask how many times the first figure of the *divisor* is contained in the first figure of the *dividend*, and find it *once*, therefore I place a 1 in the *quotient*, and multiply the whole *divisor* by it, and place the product under the two first figures of the *dividend*, and subtract it therefrom, and it will stand thus :

Second work.

$$\begin{array}{r} 15 \overline{) 19260} (1 \\ \underline{15} \end{array}$$

Remains 42

Thirdly, To this remainder 4 I bring down the next figure, *viz.* 2, (always making a dot under the figure brought down) and it is 42 ; then I ask how often the first figure 1 in the *divisor* is contained in 4, and it is 4 times ; but, upon trial, I find 4 times 15 is 60, therefore as 60 is more than 42, I must take a less figure ; I therefore make trial of a 2, and find twice 15 is 30, which I place under 42, and subtract it, and there remains 12 which stands thus :

Third work.

$$\begin{array}{r} 15 \overline{) 19260} (12 \\ \underline{15} \end{array}$$

42

30

Remains 126

Fourthly, I now make a dot under the 6, and bring down by the side of the 12, and it is 126 ; then I ask how many times 15 is contained in 126, and it is 8 times ; therefore I place 8 in the *quotient*, and multiply 15 by 8, and it is 120, which I place under 126, and subtract it, and there remains 6, which I bring down by the side of the 0, and it is 60 ; then I ask how many times 15 is contained in 60, and it is 4 times ; therefore I place 4 in the *quotient*, and multiply 15 by 4, and it is 60, which I place under 60, and subtract it, and there remains 0, which I place under the 0, and the *quotient* is 1284.

many of the first figure of the *divisor* I can have in the two first figures of the *dividend*, and find it 12 times ; but according to *note 3*, it never goes above nine times ; therefore I multiply 15 by 9, and it is 135 ; now I can't take 135 out of 126, therefore 9 times is too much, and I make trial of a less figure, to wit, 8, which I put in the *quotient*, and multiply 15 by it, which is 120, and place it under 126, and there remains 6, which stands thus

Fourth work.

$$\begin{array}{r}
 15 \overline{) 19260} (128 \\
 \underline{15 \dots} \\
 42 \\
 \underline{30} \\
 126 \\
 \underline{120} \\
 6
 \end{array}$$

Lastly, I make a dot under the cypher (0) and bring it down by the side of the 6, thus, 60 ; then I ask how many times the first figure of the *divisor* is contained in 6, or the *ones* in 6 is 6 times ; but, upon trial, I find it will go but 4 times ; therefore I place a 4 in the *quotient*, and multiplying 15 by it, find it to be 60, which I place under the other 60, and there remains 0, and the work is done, as under.

Last work.

Divisor 15)19260(1284 *quotient*

$$\begin{array}{r}
 15 \overline{) 19260} \\
 \underline{15 \dots} \\
 42 \\
 \underline{30} \\
 126 \\
 \underline{120} \\
 60 \\
 \underline{60} \\
 0
 \end{array}$$

Proof.

$$\begin{array}{r}
 1284 \text{ quotient} \\
 15 \text{ divisor} \\
 \hline
 6420 \\
 1284 \\
 \hline
 \end{array}$$

Product 19260 *dividend*

☞ *Note,*

Of DIVISION.

Note, The reason of making dots under every figure you take down, is because you may not mistake which figure comes next in course; but when you are quite perfect, you need not trouble yourself with them.

Tyro. This is a plain demonstration, and I see now the manner of working *division*, though, I own, I am not perfect yet.

Philo. I shall give you an example or two more then, and shew you at the same time how to prove it by *addition*.

EXAMPLE 2.

Divide by 375) 24172196 (64459 Answer
*2250.....

	1672		64459
	*1500		375
	1721		322295
	*1500		451213
	2219		193377
	*1875		24172125
	3446		71 add rem
	*3375		24172196 proof
Remains	*71		
Proof	24172196		

The P R O O F.

This example is proved the same as the last, by multiplying the *quotient* by the *divisor*, and afterwards adding the *remainder* to it.

P R O O F by ADDITION.

Though *multiplication* be an infallible proof of *division*, yet in large sums, *addition* is much shorter and easier, and when the work is right, is as infallible as the other. If indeed it be objected, that a boy may vary or alter the figures (as in proving *multiplication* by the cross) in order to make the sum come right, there is

the same reason to suppose (without his Master's inspection) that he may alter his *multiplication* sum, when he comes to add it up, that it may fall right with the *dividend*. Besides, as in the very work of every *Division* sum, the *divisor* is naturally multiplied into every figure of the *quotient*, what need it to be done again? And I persuade myself, that those masters, that have made use of this method of proving *Division*, will acknowledge, that it is not only much easier to the scholar, but also much more improving; because he must set his figures in order, right under one another, or else he will be puzzled to add the sum up: he cannot therefore avoid improving himself in some measure by this method.

To prove Division by Addition.

Rule. Add the remainder of all the different products of the *divisor* together, in order as they stand, and their sum will be the same as the *dividend*. Or thus: Add the remainder (if any) and all the lower lines together in order as they stand, and you will find their sum the same as the *dividend*, if the work be right.

Note. In *Ex. 2*, I have made stars (thus*) against those figures which are to be added together; which you see is the lower row of the two; for the top row is never added.

Two Examples, with their Proofs.

EXAMPLE 3.

Divide by 5267)821695074(156008 answer

*5267.....

29499

*26335

31645

*31602

43074

*42136

Remains

*938

Proof

821695074

Ex-

Of DIVISION.

EXAMPLE 4.

Divide by 9758)94076257(9640 answer

*87822...

$$\begin{array}{r}
 62542 \\
 *58548 \\
 \hline
 39945 \\
 *39032 \\
 \hline
 \end{array}$$

Remains

*9137

Proof

94076257

Here, in the first example, I take all the lower lines of the two, against which this mark (*) is placed, and beginning at the remainder, I say, 8 and 6 is 14, 4 and 1 carry 1 to 3 is 4, and 3 is 7; then 9 and 1 is 10, that is 0 and 1 carry 1 to 2 is 3, and 2 is 5; then 4 and 5 is 9; then 6 and 3 is 9, and 7 is 16, 6 and 1 carry 1 to 1 is 2, and 3 is 5, and 6 is 11, that is 1, and 1 carry 1 to 3 is 4, and 6 is 10, and 2 is 12, that is 2 and carry 1 to 2 is 3, and 5 is 8. And thus you may proceed to prove the other example.

Tyro. I confess I could not have thought it had been so easy; and it is (in effect) proving it by Multiplication, as you observed.

Philo. I am glad you understand it; and I hope you will take such care as to have no occasion to prove the work at all. Persons in business, *Tyro*, cannot go through these forms; if they suspect they have done wrong, they look over the work a second time, and that is a sufficient proof in general.

Of CONTRACTIONS.

Tyro. What is the use of contractions?

Philo. When the divisor consists of several cyphers after a figure or figures, then cut them all off, or separate them from the figures with a dash of your pen or pencil; and also remember at the same time to cut

Of DIVISION.

95

off as many cyphers or figures in the dividend; then work the sum as if such cyphers never had been there at all, and you will have the same answer.

Ex. 1.

$$\begin{array}{r} 1000 \overline{) 9250 \overline{) 000}} \\ \hline \end{array}$$

9250 *Ans.*

Ex. 2

$$\begin{array}{r} 4 \overline{) 00 \overline{) 5476 \overline{) 00}}} \\ \hline \end{array}$$

1369 *Ans.*

Ex. 3.

$$\begin{array}{r} 12 \overline{) 00 \overline{) 54762 \overline{) 00}}} \\ \hline \end{array}$$

Ans. 4563-6

Here cut off all the cyphers in the divisor, and as many cyphers in the dividend, and divide only by the single figures, and if any thing remains, I set it after it. As in example 3, there is 6 remains, which I set after the answer, thus,—6.

Tyro. This is easy enough; and now, if you please, I should be glad to know what you mean by *Division of Parts*.

SECTION I.

DIVISION of Parts.

Philo. **DIVISION** of Parts is the dividing by any two single figures in their parts, which two figures multiplied together will be equal to the *divisor*.

Tyro. I do not rightly apprehend you.

Philo. You remember, *Tyro*, in *questions of Multiplication*, that when any number was given in the table, you found two such figures, which, when multiplied together, would make that number; so here also you do the same, only with this difference, that you here divide by them, instead of multiplying. Thus, suppose I was to divide by 24, by 32, by 48, or by 72, first divide by 3, then by 8, for 3 times 8 is 24, and the last quotient is the answer. So if I divide by 48, I divide that number first by 8, and then the quotient I divide by 6, and have the proper answer, &c.

Ex-

Of DIVISION.

EXAMPLE 1..

Divide 16488 by 24. Here 6 times 4 is 24.

First by 6) $\overline{2748}$ Quotient by 6.

Then by 4) 687 Quotient by 24.

EXAMPLE 2.

Divide 126216 by 72. 6 times 12 is 72.

First by 6) $\overline{21036}$ Quotient by 6.

Then by 12) 1753 Quotient by 72.

Tyro. I understand you; this is only then the reverse of *Multiplication*.

Philo. Nothing more, as you may more plainly see by what follows.

2. Of dividing MONEY.

Division of Money is just the reverse of *Multiplication of Money*; for as in *Questions of Multiplication*, the money is increased so many times, according to the given number, so here the money is decreased, or divided into as many parts as the given number is.

It will be easier to your comprehension to take an example in *Multiplication*, and prove it by *Division*.

If 1 ell cost £. 1 - 3 - 8 what cost 40 ells?

$$\begin{array}{r}
 8 \\
 5 \\
 \hline
 40
 \end{array}
 \begin{array}{r}
 \\
 9 - 9 - 4 \\
 \\
 5 \\
 \hline
 \text{£.47 - 6 - 8}
 \end{array}
 \text{Ans.}$$

PROOF by DIVISION.

1. If 40 ells cost £.47 - 6 - 8, what cost 1 ell?

Here I divide back by the same figures I multiplied by, carrying the pounds to the shillings, and the remainder of the shillings to the pence, doing by *twelves* and by *twelves*, as in *Addition*.

by 5) £.47 - 6 - 8

Then by 8) 9 - 9 - 4 Price of 8.

£.1 - 3 - 8 Price of 1. *Ans.*

Here I divide, first, by 5, saying, the 5's in 47 is 9 times 5, and 2 over, which is £.2; this £.2 I carry to the 6s. and it is £.2 - 6, or 46s. then I say, the 5's in 46 is 9 times 5 is 45, and 1 over, that is 1 shilling, which I carry to the 8 pence, and it is 1s. 8d. or 20 pence; then I say, the 5's in 20 is 4 pence. Now I divide this £.9 - 9 - 4 by 8, saying, the 8's in 9 is *once*, and £.1 over, which I carry to the 9 shillings, and it is £.1 - 9, or 29 shillings; then I say, the 8's in 29 is 3 times 8 is 24, and 5 shillings over, which I carry to the pence, and it is 5s. 4d. or 64 pence; then I say, the 8's in 64 is 8 times. So is the price of one ell £.1 - 3 - 8; as appears also by the work of *Multiplication*.

EXAMPLE 2.

Suppose £.88 - 4 - 0 be divided among 56 poor widows, how much is the share of each?

by 7) £.88 - 4 - 0 Here 7 times 8 is 56.

by 8) 12 - 12 - 0 Share of 8.

Ans. 1 - 11 - 6 Share of each.

If you understand the nature of multiplying any sum by any figure, you see you may as readily divide it back again.

Tyro. Sir, I see the way of working it very plainly. Have you any thing farther to add?

Philo. Nothing more, as you say you are perfect; only some questions to exercise you.

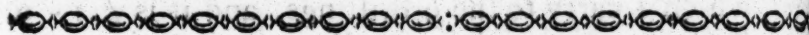
3. QUESTIONS to exercise DIVISION.

1. Divide 822306485 by 1715. *Ans.* 479479.
2. Divide 150348045 by 285. *Ans.* 527537.
3. Divide 47198714 by 6357. *Ans.* 7424, and 4346 remains.

QUESTIONS in MONEY.

1. Divide 1*l.* 16*s.* 8*d.* into 5 parts. *Ans.* 7*s.* 4*d.*
2. Divide 4*l.* 8*s.* 1*d.* $\frac{1}{2}$ into 9 parts. *Ans.* 9*s.* 9*d.* $\frac{1}{2}$.
3. Divide 25*l.* 5*s.* 4*d.* into 32 parts. *Ans.* 15*s.* 9*d.* $\frac{1}{2}$.
4. A gentleman left by will 100 pounds among three score (or 60) poor persons, to be paid every *Christmas-Day*, how much is each to receive?

I leave this last question wholly for the learner to do. And now, *Tyro*, you are come to that rule, in which you will exercise all the other *four*; therefore, if you are not perfect, you will be at a loss very often.



DIALOGUE VI.

SECTION I.

Of REDUCTION.

Tyro. **W**HAT is *Reduction*, and what does it teach?
Philo. *Reduction* is a rule compounded of all the foregoing rules, being a proper exercise for the better perfecting you in them. It teaches to reduce things of one denomination into another, and is of excellent use in common affairs of life.

Tyro. How many parts is this rule divided into?

Philo. Two. 1. *Reduction ascending.* 2. *Reduction descending.*

Tyro. What do you mean by *Reduction ascending*?

Philo. *Reduction ascending* is when things of a *smaller* denomination are reduced, or brought into things of a *greater* denomination; as farthings into pence, pence into shillings, shillings into pounds, or ounces into hundred weight, and this remember is done by *Division* only, by dividing by as many of the *less*, as make one of the *greater* denomination.

Tyro. What do you mean by *Reduction descending*?

Philo. *Descending* is just the reverse of the other; by *Reduction descending*, things of a *great* are reduced

brought

brought into things of a *less* denomination; as pounds into shillings, shillings into pence, or farthings; Cwts. into pounds, tons into quarters, miles into yards, yards into inches, and the like; and *this* remember is always done by *Multiplication* only, by multiplying the *greater* denomination by as many as it contains of the *less* denomination.

Tyro. Then I find *Reduction descending* is the easier of the two, as it is performed by *Multiplication* only.

Philo. It is so; and that is the reason it is commonly taught first.

Note, I shall prove every sum of *Reduction descending* by *Reduction ascending*, and then you will see the nature of both.

1. Of REDUCTION *descending*.

Tyro. How do you say this is performed?

Philo. By this rule. *Multiply every denomination by as many as the next less denomination contains, and you have the answer, which is called Reduction descending.* Then to prove the work, divide back by the same figures you multiplied by, and this is called *ascending*.

A few questions will make it plain to you.

Quest. 1. In 12 pounds, how many shillings?

$\text{£. } 12$
Multiply by 20 shillings in a £.

Ans. 240 shillings

P R O O F *ascending.*

In 240 shillings, how many £. sterling?
Here I divide back by 20, as before I multiplied.

$2 \overline{) 240}$ shillings

$\text{£. } 12$ *Ans.*

Here I cut off the cyphers in the *divisor* and *dividend* with a dash of my pen, and divide by 2 only, and the answer is 12.

Of REDUCTION.

Quest. 2. In £.23 - 14s. how many shillings and pence?

First by $\begin{array}{r} \text{£. } 23 - 14 \\ 20 \end{array}$ and take in the 14.

Then by $\begin{array}{r} 474 \\ 12 \end{array}$

5688 pence.

P R O O F.

In 5688 pence, how many shillings and £. *sterling*?

Divide by 12)5688 pence

Then by 2|0)47|4 shillings

£.23 - 14 as above.

Here I divide by 12, and it brings the pence into shillings; then I divide by 20, cutting off the cypher and cutting off the 4 also; then I divide by 2 only saying, the 2's in 4 is *twice*, the 2's in 7 is 3 times and 1 over, which I place before the figure 4, that also cut off, and it is 14 shillings.

This is a *general rule*; observe, that when you cut off any figure or figures, what remains in dividing must be placed before them, and is the true remainder.

Tyro. This is very plain, I own.

Philo. Then we will proceed to *question 3.*

Quest. 3. In £.45 - 17 - 6½, how many shillings, pence, and farthings?

by $\begin{array}{r} \text{£. } 45 - 17 - 6\frac{1}{2} \\ 20 \end{array}$

by $\begin{array}{r} 917 \text{ shillings} \\ 12 \end{array}$

by $\begin{array}{r} 11010 \text{ pence} \\ 4 \end{array}$

Ans. 44043 farthings.

Of REDUCTION.

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Here I multiply £.45 by 20, and take in the 17 shillings, saying, 0 is 0, but 7 is 7; then twice 5 is 10 and the 1 by the side of the 7 shillings is 11, that is 1, and I carry 1; then twice 4 is 8, and 1 is 9, which is 917 shillings. Again, I multiply the shillings by 12, and take in the odd sixpence; and *lastly*, I multiply the pence by 4, and take in the odd 3 farthings; so is the work done.

PROOF by REDUCTION ascending.

In 44043 farthings, how many pence, shillings, and pounds *sterling*?

Here I divide only back by the same figures I multiplied by, and it is done.

Divide by 4)44043 farthings

by 12)11010 pence, and 3 farthings over

by 20) 917 shillings and 6 pence over

£. 45 - 17 - 6 $\frac{3}{4}$ as above

Tyro. I understand it very well indeed.

Philo. Then I need not instruct you any more till some difficulty offers.

Quest. 4. In £. 99 - 19 - 11 $\frac{3}{4}$, how many shillings, pence, and farthings?

£. 99 - 19 - 11 $\frac{3}{4}$
Multiply by 20

1999 shillings
by 12

23999 pence
by 4

95999 farthings

Of REDUCTION.

P R O O F.

In 95999 farthings, how many pence, shillings, and pounds?

Divide by 4)95999 farthings

by 12)23999—3

by 2|0) 199|9—11

£-99 - 19 - 11½

5. In 30 guineas, how many shillings and pence?

30 guineas

21 shillings make 1 guinea.

30
60

630 shillings.

12

7560 pence.

P R O O F.

In 7560 pence, how many shillings and guineas?

12)7560 pence

21) 630(30 guineas, *Ans.*

63

00

And thus you see, *Tyro*, that any sum, of any name or denomination, may be reduced to another.

Qu

Of REDUCTION.

103

Quest. 6. In £. 472 - 15s. how many crowns, shillings, groats, and pence?

Multiply $\frac{\text{£. } 472 - 15}{4}$ the crowns in a £. and count the 15s.
for 3 crowns, saying, 4 times 2 is 8,
Crowns 1891 and 3 is 11.
5 shillings 1 crown

Shillings 9455
3 groats 1 shilling

Groats 28365
4 pence is 1 groat

Pence 113460 *Ans.*

P R O O F.

In 113460 pence, how many groats, shillings, crowns, and pounds?

Divide by 4) 113460 pence

by 3) 28365 groats

by 5) 9455 shillings

by 4) 1891

£. 472-3 crowns over, viz. 15 shillings,
as above.

Note 1. To bring pounds into pence at one operation, multiply by 240, because 240 pence make one pound. To bring pounds into farthings at one operation, multiply by 960, the farthings in a pound. On the contrary,

Note 2. To bring pence into pounds, divide by 240. To bring farthings into pounds, divide by 960.

A VOIRDUPOIS WEIGHT.

Quest. 7. In 17 C. 3 qrs. 15 lbs. how many Cwt. qrs. and lbs.

C. qr. lb.

17 - 3 - 15

by 4 qrs. make 1 Cwt.

71 qrs.

by 28 lb. 1 qr.

Here I multiply by 4, and take in the odd 3 quarters; then I multiply by 28, and take in the odd 15 pounds, and the work is done.

573

143

2003 lbs. *Ans.*

P R O O F.

In 2003 pounds, avoirdupois weight, how many quarters of hundreds, and hundred weight?

28)2003(71 qrs.

196

43

28

qrs. lb.

4)71 - 15

17 - 3 - 15 as before
C. qrs. lb.

15 lb. over.

Quest. 8. In 17 C. 3 qrs. 15 lb. how many qrs. lbs. and oz.

C. qrs. lb.

17 - 3 - 15

by 4

71 qrs.

28

573

143

2003 lb.*

16

12018

2003

32048 oz. *Ans.*

Note, The following method is very useful in many cases to reduce hundred weights into pounds, being both short and expeditious, viz.

* Set down the hundreds 4 times under one another in the following manner, and adding the odd pounds besides, gives the answer.

17 Cwt.

17

17

17

17

99 lb. in 3 qrs. 15 lb.

2003 lb.*

PROOF.

P R O O F.

In 32048 ounces, how many *lbs. grs. and Cwts.*
Divide this back again by 16, 28, and 4, and you will
have 17 *C.* 3 *grs.* 15 *lb.*

T R O Y - W E I G H T.

Quest. 9. In 5 *lb.* 2 *oz.* 14 *dwt.* of silver, how many
oz. dwt. and *grs.*

lb. oz. dwt.

5 - 2 - 14

12 *oz.* 1 *lb.* Troy by 12

62 *oz.*

20 *dwt.* 1 *oz.* by 20 and take in 14 *dwt.*

1254 *dwt.*

24 *grs.* 1 *dwt.* by 24

5016

2508

30096 grains *Ans.*

P R O O F.

Divide back by the same figures 24, 20, and 12, and
have a regard to the remainder, you'll find it 5 *lb.* 2 *oz.*
14 *dwt.* Do you now understand what has been shewn
you?

Tyro. I do very well; and I can see that *reduction*
ascending is only the proof to the other, and that they
prove one another in every respect.

Philo. If you are perfect in what I have shewn you,
there is no occasion to run through all the weights and
measures, for they are done after the same manner.

Tyro. I know it, Sir; but every example is a fresh
encouragement.

Philo. True; and I am ready to give them.

OF REDUCTION.

DRY-MEASURE.

Quest. 10. In 14 loads, 15 bushels, how many bushels and pecks?

L.	B.
	14 - 15
40 bushels 1 load,	mult. by 40, and take in the 15 bush.
<u>575</u>	
4 pecks 1 bushel	
<u>2300 pecks</u>	

P R O O F.

In 2300 pecks, how many bushels and loads?
Divide 2300 by 4, and that quotient by 40, and 15 will remain; which is 14 loads, 15 bushels.

LIQUID-MEASURE.

Quest. 11. In 12 hogheads of wine, how many gallons and pints?

12 hogheads
by 63 gallons in 1 hoghead of wine
<u>36</u>
72
<u>756 gallons</u>
by 8 pints 1 gallon
<u>6048</u> Ans.

P R O O F.

In 6048 pints, how many gallons and hogheads?
Divide this back again by 8, and by 63, and you will have just 12 hogheads.

BEER

BEER-MEASURE.

Quest. 12. In 18 butts of beer, how many hogheads and gallons?

$$\begin{array}{r}
 18 \text{ butts} \\
 \text{by } 2 \text{ hogheads } 1 \text{ butt} \\
 \hline
 36 \text{ hogheads} \\
 \text{by } 54 \text{ gallons } 1 \text{ hoghead of beer} \\
 \hline
 144 \\
 180 \\
 \hline
 1944 \text{ gallons } \textit{Ans.}
 \end{array}$$

P R O O F.

In 1944 gallons of beer, how many hogheads and butts? *Ans.* 18 butts.

CLOTH-MEASURE.

Quest. 13. In 26 yards, 3 quarters, how many quarters and nails?

$$\begin{array}{r}
 \text{Yards } \textit{qrs.} \\
 26 - 3 \\
 \text{by } 4 \text{ quarters } 1 \text{ yard, and take in the } 3 \text{ } \textit{qrs.} \\
 \hline
 107 \text{ } \textit{qrs.} \\
 \text{by } 4 \text{ nails } 1 \text{ } \textit{qr.} \\
 \hline
 428 \text{ nails.}
 \end{array}$$

P R O O F.

In 428 nails, how many quarters and yards?

Divide 428 by 4, and then by 4 again. *Ans.* 26 yards

Note. This and the two following questions are of great service in the *Rule of Three Direct*; where it is often required to reduce yards and ells into quarters, and the contrary.

Of REDUCTION.

Quest. 14. In 26 ells *English*, 3 quarters, how many quarters and nails?

$$\begin{array}{r} \text{Ells qrs.} \\ 26 - 3 \\ \text{by } 5, \text{ and take in } 3 \text{ qrs.} \quad 5 \text{ qrs. } 1 \text{ ell } \textit{English} \end{array}$$

$$\begin{array}{r} 133 \text{ qrs.} \\ \text{by } 4 \text{ nails } 1 \text{ qr.} \end{array}$$

532 nails

P R O O F.

In 532 nails, how many qrs. and ells *English*?

Divide 532 by 4, and by 5, you'll have 26 ells *English*, 3 qrs.

Quest. 15. In 26 ells *Flemish*, 2 qrs. how many quarters and nails;

$$\begin{array}{r} \text{Ells Fl. qrs.} \\ 26 - 2 \end{array}$$

$$\text{by } 3 \text{ qrs. } 1 \text{ ell } \textit{Flemish}, \text{ take in } 2 \text{ qrs.}$$

$$\begin{array}{r} 80 \text{ qrs.} \\ \text{by } 4 \text{ nails } 1 \text{ qr.} \end{array}$$

$$\text{by } 4 \text{ nails } 1 \text{ qr.}$$

320 nails. *Anf.*

P R O O F.

In 320 nails, how many quarters and ells *Flemish*?

Divide 320 by 4, and by 3, you'll have 26 ells *Flemish*, 2 qrs.

LONG

In 86

LONG-MEASURE.

Quest. 16. In 927 miles, how many yards, feet, and inches?

$$\begin{array}{r}
 \text{In 1 mile are} \quad \begin{array}{r} 927 \text{ miles} \\ 1760 \text{ yards} \end{array} \\
 \hline
 \begin{array}{r} 55620 \\ 6489 \\ 927 \end{array} \\
 \hline
 1631520 \text{ yards} \\
 3 \text{ feet } 1 \text{ yard} \\
 \hline
 4894560 \text{ feet} \\
 12 \\
 \hline
 58734720 \text{ inches}
 \end{array}$$

P R O O F.

In 58734720 inches, how many feet, yards, and miles?

Divide back by the same figures, you'll have at last 927 miles.

Note, I shall give you a fuller example in this rule, when we come to *Compound Reduction*, there being *Division* required when you are to bring rods into yards.

LAND-MEASURE.

Quest. 17. In 54 acres, how many rods and poles?

$$\begin{array}{r}
 \text{54 acres} \\
 \text{roods 1 acre, by } \begin{array}{r} 4 \text{ or multiply 54 by 160, the rods} \\ \hline 216 \end{array} \text{ in an acre, gives 8640 rods.} \\
 \text{rods 1 rood, by } \begin{array}{r} 40 \\ \hline 8640 \end{array} \\
 \text{rods } 8640
 \end{array}$$

P R O O F.

In 8640 rods or poles, how many acres?

Divide

Divide by 40, and then by 4, or divide 8640 by 160 and it gives 54 acres.

S Q U A R E - M E A S U R E .

Quest. 18. In 217 square yards, 3 feet, how many square feet, inches, and square quarters?

Square yards Feet

	217 - 5
9 square feet 1 yard, by	9
	<hr/>
	1958 square feet
144 inches 1 foot	144
	<hr/>
	7832
	7832
	1958
	<hr/>
	281952 square inches
16 square qrs. 1 inch	16
	<hr/>
	1691712
	281952
	<hr/>
	4511232 square quarters

P R O O F .

In 4511232 square quarters, how many square inches, feet, and square yards?

Divide this number back by 16, you have square inches; then by 144, you have square feet; and lastly by 9, you'll have 217 yards, 5 feet.

Note, That 12 times 144, or 1728 solid inches, make a solid foot; so that you are to multiply solid feet by 1728, to bring them into solid inches; and on the contrary, to bring solid inches into solid feet, you must divide by 1728.

TIME

Of REDUCTION.

111

T I M E.

Quest. 19. How many days, hours, minutes, and seconds are expired since the birth of our Lord and Saviour *Jesus Christ*, supposing it 1767 years, 217 days, 17 hours, and 35 minutes, and allowing just 365 days in a year.

Years Days Hours Min.

1767 - 217 - 17 - 35

365

8842

10603

5303

645172 days

24

2580695

1290345

15484145 hours

60

929048735 minutes

60

55742924100 seconds

P R O O F.

55742924100 seconds, how many minutes, hours, days, and years?

Divide this by 60, 60, 24, and 365, and you will find 1767 years, 217 days, 17 hours, 35 minutes.

Note, According to your tables in time, 365 days, 6 hours, make a year; therefore, as many years as are given, you must multiply them by 6, and add them to the hours.

Quest.

Quest. 20. If a lad be just 12 years old, how many hours are since expired?

12 years *365 <hr style="width: 50%; margin: 5px auto;"/> 4380 days 24 <hr style="width: 50%; margin: 5px auto;"/> 17520 8760 <hr style="width: 50%; margin: 5px auto;"/> 105120 hours add 72 odd hours <hr style="width: 50%; margin: 5px auto;"/> 105192	12 years 6 odd hours in 1 year <hr style="width: 50%; margin: 5px auto;"/> 72 odd hours in 12 years
--	---

* *Note,* Though I set 365 under 12, yet I multiply 365 by 12, because it is done in one line.

SECTION II.

Of COMPOUND REDUCTION.

Tyro. **W**HY do you call this *Compound Reduction*?
Philo. Because it is compounded of *Multiplication* and *Division*, and cannot be done with both.

Tyro. What is its particular use?

Philo. It teaches us to reduce any sort of foreign money into pounds *sterling*, and the contrary; so that it is useful in all business and traffic, as appears by the following table and examples.

select ALPHABETICAL TABLE of foreign Coins.

NAMES.	Value Sterling.	What Country.
shee	— 16 pence	Persia
r	— 6s. 8d.	Ditto
er	— 3 farthings	Turky
e	— 3 halfpence	Persia
an	— 3 pence	East-India
eck	— 1 penny	Muscovy
an	— 30 shillings	Japan
istiana	— 16 pence	Sweden
stake	— 1 shilling	Germany
fado	— 6s. 2d.	Ditto
o	— 2s. 10d.	Portugal
rown	— 4s. 6d.	France
o	— 5s. 3d.	Florence
o	— 7s. 6d.	Rome
o	— 1l. 2s.	Barcelona
ina	— 1l. 10s.	Aleppo
ollar	— 4s. 6d.	Italy
ion Dollar	— 4 shillings	Aleppo
rofs Dollar	— 4s. 2d.	Holland
ecie Dollar	— 5 shillings	Ditto
aland Dollar	— 3 shillings	Ditto
Philip's Dollar	— 5 shillings	Holland
old's Dollar	— 4s. 3d.	Ditto
olphus's Dollar	— 4s. 4d.	Ditto
ce of Orange's Dollar	— 4s. 4d.	Ditto
imilian's Dollar	— 4s. 5d.	Ditto
inando's Dollar	— 4s. 3d.	Ditto
ar	— 2s. 3d.	Dantzick
ar	— 2s. 3d.	Sweden
x-Dollar of the empire	— 4s. 3d. $\frac{3}{4}$	Germany
ucat	— 4s. 8d.	Hungary
o	— 4s. 8d.	Poland
lention Ducat	— 5s. 3d.	Spain
ragosa Ducat	— 5s. 6d.	Ditto
arcelona Ducat	— 6 shillings	Ditto
at	— 5 shillings	Naples
ucatoon	— 6s. 3d.	Holland
A of Eight in common	— 4s. 6d.	Spain

NAMES.

NAMES.	Sterling.	Country
A <i>Mexico</i> piece of Eight	4s. 6d. and 4s. 4d. $\frac{1}{2}$	Spain
A <i>Peru</i> piece	4s. 5d.	Ditto
A Florin	4s. 4d.	Ditto
Ditto	3s. 4d.	Germany
Ditto	2s. 6d.	Sicily
Ditto	2 shillings	Holland
Greven	1 shilling	Muscovy
Guilder, or Gilder	3s. 8d.	Germany
Ditto gold	4s. 9d.	Ditto
Ditto of <i>Noremberg</i>	7s. 1d.	Ditto
Ditto	5 shillings	Portugal
Ditto	18 pence	Dantzic
A Harpur	9 pence	Irish
A Livre is 20 Sous	18 pence	France
A Mark	13s. 4d. not current	England
Ditto	16 shillings	Denmark
Ditto	2 shillings	Germany
Mark-Lups	3s. 9d. $\frac{1}{2}$	Poland
Messe	15 pence	East-Indy
Mill-Ree (is 1000 Rees)	6s. 9d. nearly	Portugal
A Moidore, marked 4000, or 4 times 6s. 9d. viz.	27 shillings	Ditto
An Obb, or Cobb, $\frac{1}{2}$ a Harpur	4d. $\frac{1}{2}$ penny	Irish
A Pattacoon, or Pettavobn	4s. 8d.	Spain
A Ruble	10 shillings	Muscovy
Seraph	5 shillings	Turky
Timph	7 pence	Poland
Tical	2l. 16s. 3d.	China
Tari	5 pence	Sicily
Tomangold	3l. 6s. 8d.	Persia
Zachin	7s. 6d.	Venice
Zelot	7d. $\frac{1}{2}$ penny	Turky

The customary gold coins of England, are

			Dwts.	Grs.
Guinea, which when first coined weighs	5	-	9	
Half-guinea	2	-	16	$\frac{1}{2}$
Quarter-guinea	1	-	8	$\frac{1}{4}$

By an Act of Parliament, and his Majesty's Proclamation issued in *April* 1776, all gold coin must be of the weight specified in the following table, or it is not to be current, viz.

			Dwts.	Grs.
Guineas	5	-	8	
Half-guineas	2	-	16	
Quarter-guineas	1	-	8	

ro. I am obliged to you, sir.

And now, Tyro, I will give a variety sufficient for any careful learner to do any question relating to the common course of business; but before you begin, I would desire you to mind the two following observations.

Observation 1. When one sum of money, or any weight or measure, is to be brought into another sum of a different name or denomination, then always remember, that before you divide, you make the *divisor* the *dividend* of the same name. That is, if the *divisor* be pounds, pence, yards, shillings, &c. your *divisor* must also be of the same name.

Obser-

Observation 2. Take notice likewise, that your *mainder* is always of the same name as your *divisor*, be it what it will. These observations kept in memory, will be of great service to you in the *Rule of Three direct*, and in all common rules of arithmetic.

1. QUESTIONS in COMPOUND REDUCTION.

In 324 moidores, how many pounds?

324 moidores
1 moidore 27 shillings

2268
648

Shillings 210)874|8

£. 437 - 8s.

2. A merchant at London delivers to his correspondent as much broad-cloth as comes to £. 547 - 14s and he is to receive the same in cross dollars, at 40s each. How many must he receive?

Rule. Bring the money into pence, and divide the pence in 1 dollar.

s. d. £. 547 - 14 - 7
4 - 2 20
12
— 10954 shillings
50 12

Pence in a dollar 510)13145|5

Ans. 2629 cross dollars, and 5s

How
for 10

fado

7

Ans.

Quest. 4
1 rec

Rule.
de by

Of REDUCTION.

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How many crusadoes, at 6s. 2d. each, must be for 1000 guilders of *Noremberg*, at 7s. 1d. each?

s.	d.	1000 guilders	s.	d.
6	- 2	85	7	- 1
12		74)85000(1148	12	
74 pence		74	85	
		110		
		74		
		360		
		296		
		640		
		592		
		48 pence		

Ans. 1148 crusadoes and 4 shillings = 48d.

Quest. 4. How many rix-dollars, at 4s. 3d. $\frac{1}{4}$ each, will receive for £. 1750 - 15 - 6 $\frac{1}{2}$ sterling?

Rule. Bring the sterling money into farthings, and divide by the farthings in 1 dollar.

4s. 3d.

Of REDUCTION.

$$\begin{array}{r}
 s. \quad d. \\
 4 - 3\frac{1}{4} \\
 \hline
 12 \\
 \hline
 51 \\
 4 \\
 \hline
 205
 \end{array}$$

$$\begin{array}{r}
 £. 1750 - 15 - 6\frac{1}{2} \\
 20 \\
 \hline
 \end{array}$$

35015 shillings

12

420186 pence

4

Farthings in 1 dollar 205)1680746(8198 rix-doll.
 1640 and 156 farth
 3s. 3d.

407

205

2024

1845

1796

1640

by 4)156 farthings over

by 12)39 pence

3 - 3 pence

Tyro. * The remainder I see is farthings, as was the dividend; and I suppose you divide it by 4, and 12, to bring it into pence and shillings: Do you not?

Philo. You say right; and all other questions done after the same manner.

Of REDUCTION.

119

Quest. 5. In 276 crusadoes, at 6s. 2d. each, how many Hungarian ducats, at 4s. 8d. each?

s. d.	276 crusadoes	
4 - 8	74 pence in a crusado	
12		s. d.
—	1104	6 - 2
56	1932	12

168 pence in a ducat 56) 20424 (364 ducats, *Ans.* 74

362
336
264
224

Ans. 364 ducats, and 40 pence over, or 3s. 4d.

40 pence over

Quest. 6. In £. 347 - 15 - 7, how many French crowns, at 4s. 6d. $\frac{1}{8}$ each?

£. 347 - 15 - 7
20

6955 shillings
12

d.
rown 54 $\frac{1}{8}$
8

83467 pence
8 eighths in 1 penny

433 eighths

433) 667736 (1542 *Ans.* 1542 French crowns and 6d. $\frac{2}{8}$.

2347
2165

1823
1732

916
866

8) 50

6 $\frac{2}{8}$

Here I multiply the pence by 8, and the dividend and the divisor for an eighth of a penny, the remainder also is eighths of a penny; therefore, I divide it by 8, and it is 6 pence and 2-8ths over.

Note

Note 1. As you do not understand fractions as yet may not be amiss to give you a word or two by way of information. You see a *French crown* in the above example is valued at 54*d.* 1-eighth, that is, 54 pence and 1-eighth of a penny, which 1-eighth is called a fraction; the 1 which stands at top is called the numerator, and the 8 is the denominator: so also $\frac{3}{7}$ is 3-sevenths, $\frac{5}{11}$ is 5-elevenths, and $\frac{3}{5}$ is 3-fifths. Here 3, 5, and 3, are numerators, and 7, 11, and 5, are denominators.

Note 2. The Rule to work by fractions in all cases as this, is thus: Multiply the whole number by the denominator, that is, the lower figure of the fraction, and take in the top figure, or numerator.

Quest. 7. A merchant sends to his correspondent much corn as comes to £. 1575, to receive the same in ducatoons, at 6*s.* 3*d.* $\frac{3}{5}$ each; how many ought he to receive?

<i>s. d.</i> 6 - 3 $\frac{3}{5}$ <u>12</u> 75 <i>by 5, and take in the top 3</i> <u>378 fifths</u>	£. 1575 20 <hr style="width: 100px; margin: 0;"/> 31500 shillings 12 <hr style="width: 100px; margin: 0;"/> 378000 pence <i>by 5 brings d. into</i> <hr style="width: 100px; margin: 0;"/> <div style="display: flex; justify-content: space-between; align-items: center;"> <div> <i>divisor</i> 378 </div> <div style="text-align: right;"> 1890000 (5000 ducatoons 1890 <hr style="width: 100px; margin: 0;"/> 000 </div> </div> <div style="text-align: right; margin-top: 10px;"> <i>Ans.</i> </div>
---	---

Quest. 8.

anything

Quest.
 how many

Here you
 the less
 sure of

Quest. 8. In £. 1000 how many 7 pence halfpennies?

$$\begin{array}{r}
 \text{£. } 1000 \\
 \underline{20} \\
 20000 \text{ shillings} \\
 \underline{12} \\
 240000 \text{ pence} \\
 \underline{4} \\
 310)96000|0 \\
 \hline
 32000 \text{ Ans.}
 \end{array}$$

Or thus :

$$\begin{array}{r}
 960 \text{ farth.} \\
 \underline{1000} \\
 310)96000|0 \\
 \hline
 32000 \text{ Ans.}
 \end{array}$$

Quest. 9. In 100 *Portugal Pieces*, at 36 shillings each, how many pounds, half crowns, and crowns?

$$\begin{array}{r}
 \text{Piece } 36 \text{ shillings} \\
 \underline{100} \\
 210)3600 \text{ shillings} \\
 \hline
 180 \text{ pounds} \\
 8 \text{ half crowns } 1 \text{ pound} \\
 \underline{2)1440} \text{ half crowns} \\
 \hline
 720 \text{ crowns}
 \end{array}$$

Here you see I multiply, or divide always by as many the *less* as make one of the *greater*, according as the *nature* of the question requires.

Quest. 10. In 432 moidores, how many shillings, crowns, groats, and farthings?

432 moidores

27 shillings 1 moidore

3024

864

Shillings in 1 crown 5) 11664 shillings

Crowns

2332 4s. over, or 12 groats

Groats in 1 crown

15 and take in 12 groats

11662

2333

34992 groats

Farthings in 1 groat

16

209952

34992

559872 farthings

Tyro. I see now, Sir, the nature of the rule plainly but pray how do you manage when there is a sum of money to be paid in pieces of different value, and there shall be as many in number of one sort as another, that is, an equal number of every sort?

Philo. This is very useful in many respects, and done by this general rule, *viz.* Add all the pieces together, then bring the sum of money to be paid, and the sum of all the pieces, into one name or denomination; and *lastly*, divide the sum to be paid by the total of all the pieces, and the quotient is the number each, one with another.

Quest. 11. How many shillings, six-pences, four-pences, three-pences, two-pences, pence, halfpence, and farthings, of each a like number, will discharge a debt of £. 335 - 8 - 4? *Ans.* 2800 of each.

I set down all the pieces one under another, as follows:

Shilling is	12	
	6	
	4	
	3	£. 335 - 8 - 4
	2	20
	1	<hr/>
	$\frac{1}{2}$	6708 shillings
	$\frac{1}{4}$	12
	<hr/>	
pence	28 $\frac{3}{4}$	80500 pence
	4	4
	<hr/>	<hr/>
farthings	115. 115)322000	(2800 pieces of each sort
	230	
	<hr/>	
	920	
	920	
	<hr/>	
	00	

Here I bring £. 335 - 8 - 4 into farthings, and divide by 15, the farthings in all the pieces, and the answer is 2800 of each.

OBSERVATION I.

When any pieces of money are to be brought to pounds *sterling*, you should consider what relation the pieces bear to a pound *sterling*; that is, what of a pound they consist of; for by this the work will be rendered much shorter, and you will not have occasion to reduce the work into pence; but of this I shew you more fully in the *Rule of Three Direct*.

Quest. 12. In £. 270 how many nobles, at 6s. 8d. each

$$\begin{array}{r} 270 \\ 3 \text{ nobles, or 3 times 6s. 8d. is 1 pound} \\ \hline 810 \text{ Ans.} \end{array}$$

To bring nobles into pounds, divide by 3.

Quest. 13. In £. 270 how many marks at 13s. 4d.

Here, as 13s. 4d. is two-thirds of a pound, I multiply the pounds by 3, and divide by 2, as follows:

$$\begin{array}{r} 270 \\ 3 \\ \hline 2)810 \\ \hline 405 \text{ marks. Ans.} \end{array}$$

Tyro. This is much shorter than to bring the pounds and the marks into pence, and have to divide besides.

Philo. To be sure it is; and when you want to bring marks into pounds, then multiply by 2, and divide by 3; for you must have less pounds than marks because a pound is larger than a mark. And thus, by your care and observation you may work many things shorter than by the common form of rules in general. down; for it is impossible (without a large volume of rules, and even not then) to lay down rules and examples to make a careless learner understand: but a diligent learner, that is desirous to know and make himself perfect, will not only observe the rules given but will also try and contrive methods of his own, and is not satisfied with knowing things by halves.

Quest. 14. Suppose a bill of exchange was accepted London, for £. 580, for value received at *Amsterdam*, how many *Flemish* pounds ought to be remitted at *Amsterdam*?

s.	d.	
33	- 4	one F. pound
12		£. 580
<hr style="width: 50px; margin-left: 0;"/>		20
400	pence	<hr style="width: 50px; margin-left: 0;"/>
		11600
		12
<hr style="width: 50px; margin-left: 0;"/>		
		4100)1392.00

Ans. 348 *Flemish* pounds

do you understand it?

Tyro. I do; for let the question be what it will, I have nothing to do but make them both of a name, and multiply or divide by as many of the less as make up of the greater; then the quotient (if a division) will be the answer, and the remainder the same as the dividend.

Philo. Very well; then I will set you one sum more for trial.

Quest. 15. A General of an army, (consisting of 5000 men) after a very sharp engagement, lost 2380 men; coming off victorious, he, for their valiant behaviour, gave 1000 guineas to be equally divided among them, and the remainder (if any) to be given to a little hand-boy. How much did each man receive?

Tyro. I proceed thus: I take 2380 men, that were out of 5000, and there remain 2620 that escaped. Then I reduce 1000 guineas into farthings, and it 1008000, and dividing this by the number of men that escaped, viz. 2620, I find the quotient 384 farthings, that is, just eight shillings each man, and 4 farthings remaining over, which is four shillings

Of REDUCTION.

for the little errand-boy, as appears by the following work.

$$\begin{array}{r}
 5000 \text{ soldiers} \\
 2380 \text{ killed} \\
 \hline
 \text{Divisor } 2620 \text{ escaped} \\
 1000 \text{ guineas} \\
 21 \\
 \hline
 21000 \text{ shillings} \\
 12 \\
 \hline
 252000 \text{ pence} \\
 4 \\
 \hline
 \text{Men } 262 \overline{) 101008000} \text{ (384 farthings each} \\
 786 \\
 \hline
 2220 \\
 2096 \\
 \hline
 1240 \\
 1048 \\
 \hline
 4 \overline{) 192} \text{ farthings remain} \\
 12 \overline{) 48} \text{ pence} \\
 \hline
 4 \text{ shillings for the little boy.}
 \end{array}$$

Philo. Very well done indeed.

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† Na
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LONG-MEASURE.

Quest. 16. How many furlongs, rods, yards, feet, inches, and barley-corns, will reach round the world, supposing it (according to the best calculation) to be 25020 miles in circumference?

25020 miles*

8

200160 furlongs

40

8006400

11 half yards in 1 rod †

by 2)88070400 half yards

44035200 yards

3

132105600 feet

12

1585267200 inches

3

4755801600 barley-corns

* According to this calculation of 25020 miles round the globe, $69\frac{1}{2}$ miles make 1 degree. See the note in page 49, *Long-measure*.

† Note, That $5\frac{1}{2}$ yards make 1 rod; but because I cannot well multiply by $5\frac{1}{2}$, I therefore multiply by 11, and they are $\frac{1}{2}$ yards, and then divide by 2, to bring them into yards.

Another way.

As 1760 yards make a mile, if you multiply 25020 by 1760, you will have the same answer in yards; and you multiply the yards by 36, you will have the inches, &c.

SECTION III.

QUESTIONS to exercise REDUCTION.

1. **I**N £.56, how many crowns and sixpences? *Ans.* 224 crowns, and 2240 six-pences.
2. In 8568 pence, how many shillings and guineas? *Ans.* 714 shillings, and 34 guineas.
3. In £.12 - 2 shillings, how many groats, three pences, and six-pences? *Ans.* 726 groats, 968 three pences, and 484 six-pences.
4. In 750 pounds, how many moidores and guineas? *Ans.* 555 moidores, and 15 shillings, and 714 guineas and 6 shillings over.
5. In 500 *Ports*, at 36s. each, how many pounds half crowns, crowns and groats? *Ans.* £. 900, 720 half crowns, 3600 crowns, and 54,000 groats.
6. In £.500, how many pieces, at 7d. $\frac{1}{2}$, and piece at 7s. 6d. each? *Ans.* 16,000 pieces, at 7d. $\frac{1}{2}$; and 13 pieces, at 7s. 6d. each, and $\frac{1}{3}$ of a piece, or 2s. 6d.
7. A merchant at *Holland* draws a bill upon his correspondent in *London*, for 2500 ducatoons, at 6s. 2d. each, how many pounds *sterling* ought he to receive in *London*? *Ans.* £.777 - 1 - 8.

Rule. Bring a ducatoon into 5ths of a penny, (as page 138) and multiply the number of ducatoons by them then bring a pound *sterling* also into fifths, (*viz.* 12 fifths) and dividing by 1200, you will have £.777 that is, £.777 one-twelfth of a pound, or 1s. 8d.

8. A rich Nobleman has 5 villages, in every village 10 streets, in every street a dozen houses, in every house 10 rooms, in every room 2 bureaux, in every bureau 10 drawers, in every drawer 4 bags, every bag valued at 150 guineas, which he is going to exchange for £.3 - 10 - 0 shilling pieces, how many must he receive in all?

Rule.
they fol
neas.
by 72,
answer,
each.

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Rule. Multiply all the numbers by one another as they follow in order, and your last product will be guineas. Bring these guineas into shillings, and divide by 72, the shillings in a £.3 - 12 piece, you'll have the answer, which is 3780000 pieces, at £.3 - 12 shillings each.

And thus, *Tyro*, you may by diligence and observation do any sort of sum.

Tyro. Sir, I see it plainly, and will try to do them all, and prove their answers, the first opportunity.

Philo. Then I will now proceed to the most beautiful and most useful rule in all common arithmetic, viz. the *Rule of Three Direct*, commonly called the *Golden Rule of Three*.

D I A L O G U E VII.

SECTION I.

The single RULE OF THREE DIRECT, commonly called the GOLDEN RULE.

Tyro. WHAT do you mean by the *Rule of Three Direct*?

Philo. The *Rule of Three* is the *Rule of Proportion*; because it shews what relation or proportion one number bears to another.

Tyro. What is given, and what is required in the *Rule of Three*.

Philo. Three numbers are given to find a fourth, which is the answer.

Tyro. How is this fourth number found?

Philo. By this one general rule. Multiply the second number by the third, and divide by the first.

Tyro. How does the proportion of these 4 numbers stand, or what relation do they bear to each other?

Philo. As the *first* is to the *second* number, so is the *third* to the *fourth* number. For if you multiply the *fourth* number, or *answer*, by the *first*, the product will be the same as the *third*, multiplied by the *second*; but more of this by and by.

Tyro. How am I to work the *Rule of Three Direct*?

Philo. By the following rule, which you ought so far to get by heart, as to understand it without book.

A General R U L E.

1. When any 3 numbers are given, place them one after the other, in order as they stand, and this is called stating the question.

This being done, multiply the *second* by the *third*, and divide by the *first*, and you have the *fourth* number, or answer.

Let the three numbers be 4, 8, and 12.

As 4 : 8 :: 12 to a 4th number, viz. 24.

$$\begin{array}{r} 8 \\ - \\ 4 \overline{)96} \\ - \\ 24 \text{ Ans.} \end{array}$$

Thus 4 : 8 :: 12 : 24, thus read, as 4 is to 8, so is 12 to 24.

Here you see the *fourth* number, or answer, is 24.

Now, *Tyro*, it is worth your while to observe what relation or proportion these bear to each other.

OBSERVATION 1.

As the *first* is to the *second*, so is the *third* to the *fourth*. That is, as 4 is to 8, so is 12 to 24: for 4 is the half of 8, and 12 is the half of 24.

OBSERVATION 2.

As the *first* is to the *third*, so is the *second* to the *fourth*. That is, as 4 is to 12, so is 8 to 24: for 12, the *third* number, is 3 times more than 4, the *first* number; and 24, the *fourth* number, is 3 times more than 8, the *second* number.

second divided by the *second*. good, to followin

The and *third* 24 is eq

The *fourth*, is equal

The *fourth*, is equal

The *third*, d equal to

Tyro. portion h *Philo.*

will find This portion, same; b when th ards, ell effect of followin

* See dialogue

second number; or, in other words, thus: the *third* divided by the *first*, is equal to the *fourth* divided by the *second*. And thus you will find the *proportion* holds good, take the numbers which way you will, as by the following *observations*.

OBSERVATION 3.

The *first* number is equal to the product of the *second* and *third*, divided by the *fourth*; that is, 96 divided by 24 is equal to 4.

OBSERVATION 4.

The *second* is equal to the product of the *first* and *fourth*, divided by the *third*; that is, 96 divided by 12, is equal to 8.

OBSERVATION 5.

The *third* is equal to the product of the *first* and *fourth*, divided by the *second*; that is, 96 divided by 8 is equal to 12.

OBSERVATION 6.

The *fourth* is equal to the product of the *second* and *third*, divided by the *first*; that is, 96 divided by 4 is equal to 24.*

Tyro. This is very pretty indeed! and will this *proportion* hold so in any other 3 numbers?

Philo. No doubt of that; if you will try any 3, you will find they will answer to the foregoing *observations*.

This is only to shew you, *Tyro*, the nature of *proportion*, relating to numbers, without any particular name; but as for working the *Rule of Three Direct*, when the numbers are called by the names of *pounds*, *yards*, *ells*, *ounces*, *pence*, or the like; that indeed is the effect of practice, and may very soon be learned by the following *instructions*, or *directions*.

* See more of this in my *Young Algebraist's Companion*, dialogue 7, section 2.

SECTION II.

Containing some full and necessary Directions for the better performing this Rule.

Tyro. **Y**OU have already told me, that there are always *three numbers* given to find a *fourth*; but in what order am I to place them, when they are of a different name or denomination. That is, *How am I to state the question?*

Philo. Do you but mind the following instructions, and read your questions over slow and considerately, and you will soon know how to *state them*, or place the numbers in their order; for I shall first of all give you directions to work common easy sums, and then shew you how to proceed when the *first* and *third* numbers are not alike, and when there is required two or three statings.

SECTION III.

Directions to state and work easy Questions in the RULE OF THREE DIRECT.

First. **A**LWAYS remember that when three numbers are given, two of these numbers always are of one and the same name and denomination, viz. either money, weight, or measure, which *two numbers* out of the *three* must be your *first* and your *third* number, and the other in course must be placed in the middle, which *middle number* will always be of the same name as the answer. Thus, if the *second* or *middle number* be money, weight, or measure, so also will the *fourth number* or answer be.

2. If at any time the *first number*, or the *third*, be of a higher or greater denomination than the other, then make them both of one denomination by the rules of *Reduction*. Thus, if the *first number* be yards, and the *third number* ells, reduce them both into quarters, by multiplying the yards by 4, and the ells by 5; so

The RULE OF THREE DIRECT.

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6. If the *first number* be *lbs.* and the *third Cwts.* and you must reduce the *Cwts.* and *qrs.* into *lbs.* that they may not only be of one name, (weight, measure, &c.) but likewise the very same name, viz. pounds, pence, yards, quarters, &c.

3. If your *second number* be a compound quantity, such as pounds, shillings, and pence, or Cwts. qrs. and you must reduce it into pence, or into lbs. by the *Rule of Reduction*; and remember, that after you have multiplied the *second* by the *third*, and divided by your *first*, the quotient will be of the same name as you reduced your *second number* into, be it what it will.

4. In all questions in the *Rule of Three*, where there are only three numbers given, you have already been told, that *two* of those *three* will always be of one name, and must be your *first* and *third* numbers. Now, 'tis very easy to know which of those *two* must be your *first* and *third* number; for whenever a question is asked, or whatsoever number follows the question, *that* is your *third* number. Thus, what cost 24 yards? or how much does 96 lb. cost? Here 24, or 96, will be your *third* number; and so for any other question, as you will see by the two first examples, as follows:

Quest. 1. If 3 yards cost 7 shillings, what cost 24 yards?

Ans. *shill.* *ys.*

3 — 7 — 24

7 multiply

divide by 3) 168

by 210) 5'6 shillings

21. 16s. *Ans.*

Proof, or proportion.

As 3 : 7 :: 24 : 56. For 3 times 56 is 168, and 7 times 24 is 168. And thus for any other question.

Quest.

Quest. 2. If 8 ells cost 11 shillings, what cost 96 ells?

Ells	shill.	ells
If 8	— 11 —	96
		11

8) 1056

210) 1312

£. 6 - 12 *Ans.*

Note. Here you see that the question is asked: What cost 24 yards, or what cost 96 ells? therefore you may be sure these are to be your third number. Then your *first* is plainly known by being of the same name; and your *middle*, or *second* number, must be known of course, which will always be the same name as the answer. See *observation 1.*

Rule. Having stated the question, I multiply the *second* by the *third* number, or the *third* by the *second*, and divide by the *first*, and the answer is 56 shillings, which is the same name as the second number; then divide those shillings by 20, and have the answer. The second question is done the same.

Tyro. I see the nature of it plainly; but there is no way to prove sums in the *Rule of Three*, by stating the question backwards, is there not?

Philo. Yes, and a very good method it is to confirm you in the truth of what you have done; for the question will then be turned quite the reverse way.

First Proof, or Variation.

If 24 yards cost 56 shillings, what cost 3 yards?

Yards	shill.	yards
If 24	— 56 —	3
		3

24) 168 (7 shillings. *Ans.*

168

0

Here you see the *fourth* number in the original question is the *first* number in this question; the answer to the original is the *second* number; and the *second* number in the original is the answer here.

Second Proof, or Variation.

If 7 shillings buy 3 yards, how many can I have for 56 shillings?

Here I find two numbers out of the three are money, the number following the question (*how many*) is shillings; therefore 56 shillings is my third number, which is thus stated.

Shill. yards shill.

If 7 — 3 — 56

3

7)168

24 yards. *Ans.*

So that you see, by minding which *two* of the *three* of the same name, it is no difficult matter to state any common question, and others more difficult will naturally follow by use and practice.

Syro. I understand it very well indeed, and heartily thank you.

Philo. Then I shall proceed to give you more useful examples, and when any thing occurs that you don't perfectly apprehend, ask me, or read the instructions over again.

Quest.

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Quest. 3. If 3^{lb.} of tobacco (or any other thing) cost 2s. 9d. what cost 255 lb.

lb.	s.	d.	lb.
If 3	— 2	— 9	— 255
	12		33
	—		—
	33		765
			765

second number

$$\begin{array}{r} 3 \overline{)8415} \end{array}$$

$$12 \overline{)2805} \text{ pence}$$

$$20 \overline{)23} 3 - 9$$

$$11 - 13 - 9$$


P R O O F.

Pence	lb.	pence
If 2805	— 255	— 33
	33	
	—	
	765	
	765	

$$2805 \overline{)8415} (3 \text{ lb. } \textit{Ans.}$$

$$\begin{array}{r} 8415 \\ \hline 0 \end{array}$$

0

 The *second numbers* being *shillings* and *pence*, bring them into pence, by multiplying by 12, and taking in the odd 9 pence; and after I have multiplied the *third number* 255 by 33, and divided by my *first*, which is 3, the quotient is of the same name still, *viz.* pence, which I divide by 12, to bring into shillings, and 20 to bring them into pounds, as above; and thus for any common sum. See *observation 2.*

Quest.

The RULE OF THREE DIRECT.

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Quest. 4. If an ounce of tobacco cost 5 farthings,
at cost 2 C. 3 qrs. 21 lb.

oz. farth. C. qrs. lbs.
If * 1 — 5 — 2 — 3 — 21

4
—
11 qrs.
28
—
89
24
—
329 lbs.
16
—

1974
329
—

5264 oz.

5 second number

4) 26320 farthings

12) 6580 pence

2|0) 54|8 - 4d.

£. 27 - 8 - 4 Ans.

* Note, Because the
first number is ounces,
I bring the third num-
ber also into ounces,
to answer it; but after
I have multiplied by
my second number 5,
they are all farthings;
and as my first num-
ber is only an unit (or
1) I do not divide by
that; for 1 neither mul-
tiplies nor divides: I
therefore only divide
the farthings by 4, 12,
and 20, and the ques-
tion is done. See the
proof of the next ques-
tion.

Quest.

Quest. 5. If 2 C. 3 qrs. 21 lb. (or 5264 ounces) cost £. 27 - 8 - 4, what cost an ounce?

oz. £. s. d. oz.
If 5264 — 27 - 8 - 4 — 1

20

548

12

6580

4

5264) 26320 (5 farth. *Ans.*
26320

0

Quest. 6. Bought a silver cup and tankard, which weighed 3 lb. 11 oz. troy, and cost me £. 8 - 18 - what did they lie me in per ounce?

Here, as the question is, *What did they cost per ounce*, you must know your answer is to be in money; for you must your *second* number be, and the other two numbers will naturally follow, thus:

lbs. oz. £. s. d. oz.
If 3 - 11 — 8 - 18 - 2½ — 1

120z. 1 lb. troy 20

47 oz.

178

12

2138 pence

4

47) 8554 (182 farthings. *Ans.*

47

385

376

94

94

0

4) 182 farthings

12) 45 - 2

3s. 9d. ½ per oz. *Ans.*

Take notice that whenever your first number is large, and the *third* small (as in this example) that you always reduce the *middle* or *second* number to its lowest denomination.

Quest.

m in c
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ery year

Rule.

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Year

Yearly

Estate

He loses

Quest. 8.

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loads i

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Rule. F

guinea,

at 9 gui

Quest. 7. A gentleman has an estate which brings in clear £.2000 a year, and the charges he is at, the day with another, for housekeeping, and other expences, are £.5 - 15s. I demand what he loses or saves every year?

Rule. See what £. 5 - 15s. *per* day amounts to in a year, and take it out of his yearly income £.2000; this shews what he saves; but if his expences be more, take 2000 from that, and the remainder is what he loses.

Day	£.	s.	Days
If 1	5	15	365
	20		115

115	1825
	4015

210)419715 shillings

Yearly expences	£.2098 - 15	
Yearly expences	_____	£.2098 - 15
Estate	_____	2000 -

He loses yearly	_____	£.98 - 15 <i>Ans.</i>

Quest. 8. A farmer agreed with his servant to thresh the corn he had, and the servant was to receive a pence for every 7 quarters; now, he threshed in all 15 loads 1 quarter, and has received of his master, at different times, by cash and goods, 9 guineas; I demand how the reckoning stands between them?

Rule. First, see what 15 loads 1 quarter come to, at 1 pence, or 21 shillings per quarter, and then subtract 9 guineas out of it.

grs. shill. loads gr.
If 7 — 21 — 15 — 1

5 qrs. 1 load

76 qrs.

21

76

152

7) 1596

2(c) 22|8 shillings

£. 11 - 8 Ans.

Servant's wages for threshing

£. 11 - 8

He received 9 guineas, viz.

— 9 - 9

Due to the servant

£. 1 - 19

Quest. 9. A corn-factor sends to his correspondent in Spain, 10,000 quarters of wheat, and agreed £. 8 - 7 - 6 per load, and he has received by several remittances £. 10,000; what is still due to him?

Load £. s. d. qrs.
If 1 — 8 - 7 - 6 — 10,000

5

20

5 qrs. 167

12

Wheat
Received

£. 16
100

Due to him

£. 6

2010 pence

10000

First number 5) 20100000

12) 4020000 pence

2) 335000 shillings

£. 16750 Ans.

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Ex. 10. Bought 4 bags of hops, which weighed as under

C. qrs. lb.

No. 1 — 2 — 3 — 11

2 — 1 — 2 — 27

3 — 2 — 1 — 17

4 — 1 — 3 — 15

And gave after the rate of 5 guineas *per cwt.* what do they come to?

8 — 3 — 14

C.	£.	s.	C. qrs. lb.
1	5	5	8 — 3 — 14 * *
2 lb.	20		4

2 lb.	105	35 qrs.
		28

284
71
994 lbs.
105

4970
9940

First number 112) 104370 (931 shillings divided by 20, is 46 - 11. So that the answer is 46 - 11 - 10½.

1008
357
336

210
112

98 remainder
12

112) 1176 (10 pence
112

56 remainder
4

112) 224 (2 farthings
224
0

* * Note,

Note, I reduce the first and third into *lbs.* and the second into shillings, and the quotient is shillings and the remainder 98 I multiply by 12, and divide by the same divisor, (*viz.* 112) and it gives 10 pence in the quotient; and the next remainder I multiply by 4, and it gives 2 farthings.

Note See a shorter way than either this, or page 104, to bring *cwts. grs. and lbs.* into *lbs. viz.* in example 1, in Tare and Tret.

Quest. 10. There is a ship worth £.2000, of which I have the $\frac{7}{32}$ parts, I demand what my share is worth.

First, according to the common order of the Rule of Three, I say,

Parts. If 32 be — £.2000, what is 7 *Parts*

$$\begin{array}{r} 7 \\ \hline 32 \overline{) 14000} (437 \text{ pounds} \\ 128 \end{array}$$

120

96

240

224

16 remainder

20 shillings

$$\begin{array}{r} 32 \overline{) 320} (10 \text{ shillings} \\ 0 \end{array}$$

Ans. £. 437 -

Note, I multiply the remainder 16 by 20, and divide by the same *divisor* 32, and the quotient is shillings. Had any thing remained, I should now have multiplied it by 12, and divided by 32, and the quotient would have been pence, &c.

Note, It often happens, that dividing by the parts both shorter and easier than dividing by the whole : being but practice and observation can make a man master of this ; for ten thousand rules without these are very little account.

OBSERVATION.

Here, according to the nature of questions of *Multiplication* or *Division of Money*, I perform the above question with ease.

Parts		Parts
If 32	£. 2000	7
	7	

times 4 is 32 by 8) 14000

4) 1750

Ans. £. 437 - 10

Here I divide by 8, and by 4 ; when I divide by 4, and 2 remain, which I call 2 pounds : then I say, the part of £. 2 is 10 shillings. — And now you may see beauty of *Multiplication* and *Division of Money*, if you do but observe it well.

Ans. I humbly thank you, sir, and see there is nothing like knowing the former rules perfectly.

Obilo. As I said before, many questions may be performed by *Multiplication* and *Division of Money*, if they are rightly applied, and much shorter and easier than the common order of the *Rule of Three*. I shall give you an instance, and no more, which, if well observed, will be sufficient ; for according to the nature of the question, a great many figures may be saved, as well as trouble.

Quest.

Quest. 11. Bought a box of superfine tea, which weighed 2 qrs. 7 lb. and gave £. 73 - 12 - 7½ for what is 5 lb. of it worth?

First, and common Way.

qrs. lb.	£.	s.	d.	lb.
If 2 - 7	73	12	7½	5
28	20			
<hr/>				
63	1472 shillings			
	12			
<hr/>				
	17671 pence			
	4			
<hr/>				
	70686 qrs.			
	5 third number			
<hr/>				
63) 353430 (5610 farthings				
	315			
	<hr/>			
	384			
	378			
	<hr/>			
	63			
	63			
	<hr/>			
	0			
	Farthings			
	4) 5610			
	<hr/>			
	12) 1402 - ½			
	<hr/>			
	2) 1116 - 10			
	<hr/>			
	<i>Ans.</i> 5-16-10½			

Short Way, by Multiplication and Division of Money.

lb.	£.	s.	d.	lb.
If 63	73	12	7½	5
			5	
<hr/>				
63 is 9 times	7) 368	3	1½	
	<hr/>			
	9) 52	11	10½	
	<hr/>			
	5 - 16 - 10½ <i>Ans.</i>			

Here you see that the second way has but about half the figures.

Tyro. I see it very plainly; and I think I am able to find any common sum of but one *flating*.

Philo. Then we will proceed to *Section 4*.

SECTION IV.

Containing some necessary Directions in more difficult Questions, and such as require more than one flating.

Q. HOW am I to manage when the *first* and *third* numbers are not of one name or denomination?

Philo. Bring them into one denomination (according to *direction 2, section 1*) and proceed by multiplying the *second* by the *third*, and divide by your *first*, as before.

Quest. 12. If 3 yards cost 8s. 3d. what cost 96 ells English?

Yards	s.	d.	Ells Engl.
If 3	8	3	96
4	12		5
—	—		—
12	99		480
			99
			—
			4320
			4320

First number 12)47520

12) 3960 pence.

210) 3310

£. 16 - 10 Ans.

Rule. Here, because the *first* number is yards, and the *second* ells *English*, I bring them both into quarters, by multiplying the yards by 4, and the ells by 5, and then proceed as before.

First Proof, or Variation.

Quest. 13. If 96 ells *English* cost £. 16 - 10s. what cost 3 yards?

<i>Ells</i>	<i>£.</i>	<i>s.</i>	<i>Yards.</i>
If 96	16	10	3
5	20		4
480 qrs.	330 shillings	12 qrs.	
	12		
	3960 pence		
	12 third number		

First number 480) 47520 (99 pence. *Ans.* or 8s. 3d.

$$\begin{array}{r}
 4320 \\
 \hline
 4320 \\
 \hline
 4320 \\
 \hline
 0
 \end{array}$$

☞ Here you see the *fourth* number, or answer, is same as the *second* number in the last question. See next question.

Second Variation, or Proof.

Quest. 14. If I buy 3 yards of Holland for 8s. 3d. how many ells *English* may I have for £. 16 - 10?

Here, because the answer is to be in measure, therefore measure must be my second number, thus:

s. d. Yards	£. s.
If 8 - 3 — 3 —	16 - 10
12	20
—	—
99 pence	330 shillings
	12
	—
	3960 pence
	3 second number
	—

First number 99) 11880 (120 yards

99	Yards.
—	120
198	4 qrs. 1 yard
198	—
—	—
Qrs. 1 ell 5) 480 qrs.	
—	
96 ells. <i>Ans.</i>	

Here you see I do not bring the second number, into quarters, as I did before in the other two questions, for it is better to work with it as it is; because it saves a great many figures, and the answer will be 120 yards, which I reduce into ells very easily, by multiplying by 4, and dividing by 5; and if you consider all the three different ways of working this question, it will be of great service to you.

Quest. 15. A draper bought 12 pieces of Holland, each piece containing 23 ells *English*, and gave after the rate of 3s. 4d. per ell *Flemish*; what did they come to?

First, 23 ells 1 piece
12 pieces

276 ells in all.

<i>Ell Flem.</i>	<i>s.</i>	<i>d.</i>	<i>Ells Eng.</i>
If 1 ———	3	- 4	276
3 qrs. 1 ell <i>Fl.</i> 12	12		5
3		40 pence	1380 qrs.
			40
			3)55200
			12)18400 pence
			210) 15313 - 4
			£. 76 - 13 - 4 <i>Ans.</i>

Tyro. I am obliged to you; and now, if you please I will try at one.

Philo. With all my heart; I like to see you in the mind.

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Quest. 16. A woollen-draper bought 4 packs of cloth, each pack containing 3 parcels, each parcel 6 pieces, and each piece 40 yards, and gave 3*l.* 10*s.* for every 6 yards, one with another; what do they come to?

Tyro. First, I see how many yards there are in all, by multiplying them together as they stand, thus:

4 packs	<i>Yards</i> £.	<i>s.</i>	<i>Yards.</i>
3	If 6	3	10
	20		70
12 parcels			
6	70	6	201600 shillings
72 pieces			
40	2	0	3360
			1680 <i>Ans.</i>

2880 yards in all.

Philo. Very right; and you may prove this sum several ways at leisure, which will be of service. First, 3 - 10*s.* every 6 yards is 11*s.* 8*d.* for 1 yard. Therefore say, if 1 yard cost 11*s.* 8*d.* what cost 2880, and it will be the same answer. And again, you know 80 yards, at £.1 per yard, is £.2880; therefore say, 20 shillings be 11*s.* 8*d.* what will 2880 be; and you will have 1680 for answer, as before.

Quest. 17. How many dozen of candles, at 5s. 3d. per dozen, may I have in exchange for 3 pieces of Irish, each 20 ells long, at 3s. 9d. per yard?

Here are two statings in this question; first, see what the *Irish* comes to, and as the question asks, how many dozen of candles, therefore 1 dozen must be the middle number of the second stating.

First, 3 pieces, each 20 ells, is 60 ells in all.

Yard	s.	d.	Ells	s.	d.	Doz.	£.	s.	d.
If 1	3	9	60	If 5	3	be 1	14	1	3
4	12		5	12			20		
4	45		300	63			281		
	300						12		
4	13500					63	3375	(53 dozen	
							315		
12	3375	pence					225		
210	281	1 - 3					189		
							36		
		14 - 1 - 3 Irish cost					12 in 1 doz.		
						63	432	(6 candles	
							378		
							54		

Answer 53 dozen and 6, that is, 53 dozen and a half the remainder 54, and all other remainders, are placed fraction-wise over the *divisor*, thus, $\frac{54}{6}$. So that the true answer is 53 dozen and 6 candles, and 54 sixths, or three parts of another candle, or $\frac{3}{2}$, which wants but $\frac{1}{2}$ of another whole one.

Tyro. I understand it all but the $\frac{6}{7}$, how do you make that out?

Philo. You will see better when you come to *Vulgar Fractions*; for $\frac{6}{7}$ is the same as $\frac{54}{63}$. For divide the

erator 54 by 9, and it is 6, for a *new numerator*, and divide 63, the *denominator*, also by 9, and it is 7. But the answer without the fraction is near enough, and you must not mind these curiosities at present.

Quest. 18. A woollen-draper bought a certain quantity of serge and broad-cloth for £. 500: the quantity of broad-cloth he bought was 500 yards, at 12s. 6d. per yard; now he bought 3 times as many yards of serge as he did broad-cloth. I demand how much each cost, and what the serge cost *per yard*?

First, I find what the broad-cloth comes to.

Yard	s.	d.	Yards	
1	12	6	500	Both of them cost £. 500 -
	12			Broad-cloth cost 312 - 10
				<hr/>
	150			The serge cost 187 - 10
	500			
				<hr/>
	12	75	000	
				<hr/>
	2	0	625	0 shill.
				<hr/>

£. 312 - 10 *Ans.* broad-cloth

Here you see the broad-cloth cost in all £. 312 - 10s. which taken out of £. 500 (what they both cost) the serge then must cost in all £. 187 - 10s. Now the question says, he had three times as many yards of serge as he had of broad cloth, which is in all 1500 yards that is, 3 times 500) of serge. Then to find how much the serge cost *per yard*, I say,

The RULE OF THREE DIRECT.

Yards £. s. Yard.
If 1500 cost 187 - 10 — 1

20

3750

12

First number 15|00)450|00(30 pence, or 2s. 6d. *Ans.*
45 the serge cost per yard.

0

Tyro. Sir, I am highly obliged to you; and I perceive, that it is nothing else but considering the nature of the question, and two statings are as easily done as one. But pray, if I may be so free, is not Interest carried up, or done by the *Rule of Three*?

Philo. There is a shorter way of doing it; but it may be done very easily; as also, *exchange, loss and gain, barter, fellowship*, and many such like rules, tho' they bear those different names, are all but the *Rule of Proportion*; however, to satisfy you, I will give you an example of two of each, to prepare you the better to work them.

1. An EXAMPLE in INTEREST.

Quest. 19. What is the interest of £. 360, for three years, at £. 3½ (or £. 3 - 10s.) *per cent. per annum*?

Note. Remember that *per cent.* signifies £. 100; that in other words it is thus: What comes the interest of £. 360 to for three years, at £. 3½ for every £. 100. Say,

If £. 100 — £. 3 - 10 — £. 360
20 70

70 1|00)252|00

2|0)25|2 shillings

£. 12 - 12 for a year. *Ans.*
3 years multiply

£. 37 - 16 for 3 years. *Ans.*

And a
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9 days,
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First,
£. £
100—

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ncipal
9 days

Ans.

An

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And after this manner any sum, at any rate *per cent.* may be done.

Note, When there are odd months, you must allow proportion, thus: Suppose it had been six months more, you see one year, or twelve months, is £.12 - 12s. therefore, six months is half of it, viz. £.6 - 6s. and three months is £.3 - 3s. more.

Quest. 20. A person left his niece £.650, to be paid her when she came to proper age, with interest upon it at 4 *per cent.* Now it lay in the executor's hands 219 days, what had she got to receive, principal and interest together?

First, I find the interest for one year.

£.	£.	£.	Days	£.	Days.
100	— 4 —	650	If 365 be 26 —		219
		4			26
		<u>£.26</u>			<u>1314</u>
					438

cut off 2 cyphers, which is the same as if divided by 100, and the answer is £.26 for one year's interest.

365) 5694 (15 £.
	365
	<u>2044</u>
	1825
	<u>219</u>
	20

Principal — £.650 -
219 days interest is 15 - 12

Ans. She has £.665 - 12 to receive.

) 4380 (12 s.
365
<u>730</u>
730

2. An EXAMPLE in bartering, or exchanging. See Question 17.

Quest. 21. Two merchants, *A* and *B*, barter, or exchange with one another in traffic. *A* has sugar, which he sells at £.4 per cent. for ready money; but in barter he will have £.4 - 13 - 4. *B* has wine at £.13 per hoghead; now the question is, how much *B* ought to advance his wine in barter, to equal the advance of *A*'s sugar.

First, sugar — £.4 - 13 - 4 in barter
 For ready money — 4 - -
 —————
 £. - 13 - 4 advance

Then, If £. s. d. £.
 If 4 advance 13 - 4 — 13
 12

160

13

480

160

First number 4)2080

12) 520 pence

2|0) 4|3 - 4

£. 2 - 3 - 4 Ans.

Ans. *B* must advance his wine £. 2 - 3 - 4: that in barter his wine worth £. 15 - 3 - 4 hoghead.

3. EXAMPLES in Profit and Loss, or Loss and Gain.

Quest. 22. Bought 3 pipes of wine for 100 guineas, one of which leaked or ran out 23 gallons; the remainder I sold at the rate of 3 half crowns per gallon, what was my gain or loss?

First, 1 pipe is 126 gallons
 3

 3 pipes are 378 gallons
 23 leaked out

 355 gallons left

Gall. s. d. Galls.
 If 1—7 6—355
 12 90

 90 12)31950

 210)26612-6

 Sold for £. 133-2-6

Made of the whole £. 133 - 2 - 6

Cost 100 guineas, viz. 105 - -

Gain ——— £. 28 - 2 - 6 *Ans.*

Quest. 23. What is gained *per cent.* that is, what is gained in laying out £. 100, when 1s. brings in 14d. $\frac{3}{4}$?

First, 14d. $\frac{3}{4}$ d. d. £.
 shilling 12 If 12 gain $2\frac{3}{4}$ —100

 shilling gains $2\frac{3}{4}$ 4 20

 11 2000
 12

24000 pence

11

12)264000

4)22000

12)5500

210)4518-4

Gained *Ans.* £. 22 - 18 - 4

Quest. 24. A gentleman that saw very plainly he could not live upon the common interest of his money, set up as a common brewer, and laid out £. 1000 for that purpose, in fitting up an office, and buying utensils; the hops, malt, excise, wages, and other charges at the year's end amounted to £. 400 more; in which time he sold out as much beer as came to £. 6875. I demand what he gained in all, and how much he gained *per cent*.

First, Beer amounted to £. 6875
His charges are 5000

Gained in one year 1875

If £. 5000 gain 1875 — £. 100
100

* 5|000) 187|500

£. 37 - 2500 remains
20

5|000) 50|000

10 shillings

Ans. He gained in all £. 1875, which is £. 37 - 10 *per cent*

* *Note,* I cut off three cyphers in the *divisor*, and three figures in the *dividend* to answer them; then divide 187 by 5, and it is 37, and 2 over, which I place before the 500 that I cut off, and it makes 2500 remains, which I multiply by 20, and cut off three cyphers, and divide by 5 only, gives 10 shillings.

4. EXAMPLES in COMPANY called FELLOWSHIP

Quest. 25. Two tradesmen or merchants, *A* and *B* enter into partnership, and purchase one common stock. *A* put in £. 1750, and *B* put in £. 1050, and they

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first year of trading £.800 clear; what is the share each, according to the sum each advanced at first?

Rule. In all questions of this nature the rule is, add together what every person first put into stock, then say, as the *whole stock* is to the *gain or loss*, so is what every person separately put in, to his part of *gain or loss*. That is, the *whole stock* added together is your *first* number; the *gain or loss* your *second* number; and what every man separately put in is your *third* number; and the different answers, or *fourth* number, will be every man's proper share.

The Work.

A put in	—	—	£. 1750
B put in	—	—	1050
			<hr/>
Whole stock	—		£. 2800

£. 2800 gain £. 800 — £. 1750 A's stock
800

28!00)14000!00(500£. A's gain.
140
—
00


Secondly, If 2800 gain 800 — 1050 B's stock.
800

28!00)8400!00(300£. B's
84 [gain.
—
00

Ans. A's share of the gain is — £. 500
B's share is — 300

Gain — — 800 proof

✍ Note,

 *Note*, If the *gain* or *loss* be pounds, shillings and pence, reduce it accordingly, and your answer will be of the same name; and if there be remains, add them together, and they will be always equal to the *divisor* if the work be right, and you must carry one of the next denomination when it happens so.

Quest. 26. A person broke, in debt, for £. 1890; he gave up all his effects to his creditors, which amounted to £. 661 - 10s. how much then did each creditor receive in the pound?

Note, Had each person's debt been specified, there must have been as many different statings, but in this case but one, thus:

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{s.} \quad \text{£.} \\ \text{If 1890 be 661 - 10 — 1} \\ \quad \quad \quad 20 \end{array}$$

$$\begin{array}{r} 1890 \overline{) 13230} \text{ (7 shillings in the pound. Answer)} \\ \underline{13230} \\ 0 \end{array}$$

P R O O F.

$$\begin{array}{r} \text{Debt } \text{£. 1890} \\ \quad \quad \quad 7 \\ \hline 210 \overline{) 13230} \end{array}$$

£. 661 - 10 effects.

Now, suppose, for example's sake, that any creditor's debt or demand was £. 58, what must he receive? Say, if £. 1 be 7 shillings, what will £. 58 and you will find it £. 20 - 6 for his part; and for the others, be they more or less.

Tyro. I am extremely obliged to you for your explaining things to me in this manner.

Philo. I have no other aim but your advantage; nor could I have been so long and particular, but that your knowledge in this *rule* is the foundation of all the other rules, and even of the *mathematics* itself. Thousands of examples more, 'tis true, might be given you; but as they depend upon what you have seen, and your constant practice, I shall give you but one more, which some measure varies from the former ones.

Quest. 27. A woollen-draper bought of a clothier, an equal quantity of four sorts of cloth, which cost in £. 367 - 10s. Some cost 18s. 6d. some 14s. 3d. some 11s. 6d. and some 4s. 9d. *per yard*: how many yards of each had he?

Rule. The method of working all sums of this sort is this: add all the different prices together into one sum, which is the *first* number; then 1 yard, or 1 *lb.* will be your *second* number, and the whole sum will be out your *third* number.

The Work.

I add all the different prices, thus:

s.	d.
18	6
14	3
11	6
4	9
<hr/>	
£. 2	9 -

s.	Yard	£.	s.
9	1	367	10
		20	
		<hr/>	
9	1	49)7350	(150 yards of each
		49	<i>Ans.</i>
		<hr/>	
		245	
		245	
		<hr/>	
		0	

Tyro.

Tyro, I perceive the nature of it plainly.

Philo. Indeed, *Tyro*, if you understand these examples, you are able to solve any common question relating to business.

N. B. My intent, *Tyro*, was to have given you here notion of *timber-measure*, and how to *gauge* a common cask, cooler, or piece of malt, &c. or to measure any regular piece of ground; it being not only diverting but also useful in the country, and very satisfactory to parents, when their children have some knowledge of these things. But I shall reserve this for a dialogue by itself, and only leave you a few questions for practice.

QUESTIONS to exercise the Learner in the RULE OF THREE.

28. The rents of a whole parish amount to £. 175 and a rate is granted of £. 32 - 16 - 3; what is that the pound? *Ans.* 4d. $\frac{1}{2}$.

29. A bankrupt is indebted £. 2980 - 10s. but all his effects amount but to £. 931 - 8 - $1\frac{1}{2}$; what have his creditors in the pound? *Ans.* 6s. 3d.

30. Bought a lighter of coals, containing 33 chaldrons, 12 bushels, for which I gave £. 50; what they lie me in *per bushel*? *Ans.* 10d.

31. Bought of a goldsmith 4lb. 11oz. 10 dwts. plate, at 5s. 4d. *per ounce*; what does it come to? *Ans.* £. 15 - 17 - 4.

32. What is the interest of £. 631 - 5s. for a year, 3 *per cent*? *Ans.* £. 18 - 18 - 9.

33. What comes the commission of £. 245 - 6s. to $2\frac{1}{2}$ *per cent*? *Ans.* £. 6 - 2 - $7\frac{1}{4}$, $\frac{1}{2}$.

34. What must I give for the $\frac{6}{32}$ parts of a ship, that is worth £. 635 - 5s. *Ans.* £. 119 - 2 - $2\frac{1}{4}$.

35. Shipped for *Barbadoes* 500 pair of stockings, 3s. 6d. *per pair*, and 1650 yards of baize, at 15d. *per yard*; and I have received in return 348 gallons rum, at 6s. 8d. *per gallon*, and 750 lb. of indigo.

6d. per lb. what remains due upon my adventure?
 $\text{£. } 18 - 7 - 6.$

Rule. See what the stockings and baize come to, add them together; then see what the rum and in both come to, and subtract it from the other, then you will find the answer as above.

6. How many bricks, 9 inches long, and 4 inches wide, will floor a room 18 feet square; that is, 18 feet square, and 18 feet long.

Rule. First, find the content of the room in square inches, that is, multiply the length by the breadth; then multiply that product by 144, which will bring it into square inches: this done, multiply the length of the brick by the breadth, and you will have 36 inches, which is your divisor, and the answer is 1296 bricks.

7. How many 10 inch tiles will floor a malt kiln 12 feet long, and 14 feet wide. *Ans.* $322 \frac{56}{100}.$

Rule. Multiply 10 by itself, gives the contents of one tile in square inches; then find the contents of the kiln in inches, and divide by 100, it is 322, and better a half.

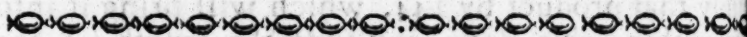
8. Bought a ton of iron, which cost $\text{£. } 19 - 10s.$ being in number 32 bars: I desire to know the price of each, and what they weighed one with another? *Ans.* $2s. 2d. \frac{1}{4},$ and each bar weighs $2 \text{ qrs. } 14 \text{ lb.}$

9. There is a cistern that holds 4 hogheads of water (allowing 63 gallons to the hoghead) in which are placed two pipes, the larger of which will discharge 100 gallons every minute; but if they both be set open, they will discharge $2 \frac{1}{2}$ gallons every minute: I demand how long time the cistern will be filled by each pipe severally, and in how long when they are both set open together. *Ans.* Greater pipe will fill it in 2 hours 48 minutes; the small one in 5 hours 36 minutes; and both in 1 hour 52 minutes.

10. There is a steeple, which stands right upon level ground, whose shadow measures 75 yards, at the same time that the shadow of a strait walking-stick, which

which is a yard long, measures 5 feet: I demand height of the steeple? *Ans.* 45 yards.

41. As I was beating on the forest grounds,
Up starts a hare before my two greyhounds;
The dogs being light of foot, did fairly run
Unto her 15 rods, just 21.
The distance that she started up before
Was 4 score 16 rods just, and no more:
Now this I'd have you unto me declare,
How far they ran before they caught the hare?
Ans. 336 rods.



DIALOGUE VIII.

SECTION I.

The RULE OF THREE INVERSE

Or, RECIPROCAL PROPORTION.

Tyro. WHAT do you mean by the Rule of Inverse?

Philo. It is often called the Rule of Three Reciprocal; that is, it is contrary to the Rule of Three Direct. as in the Rule of Three Direct you multiply your first number by your third, and divide by your second; you multiply your second by your first, or your first by your second, and divide by your third, and the third number is the answer.

Note, Your third and first number must be of the same name or denomination as in the Rule of Three Direct, and the fourth number, or answer, will be of the same name as the second number.

The RULE OF THREE INVERSE.

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quest. 1. How many yards of shalloon, 3 qrs. wide, line 4 yards of cloth, which is 5 qrs. wide? *Ans.* yards.

is in the *Rule of Three Direct*, so here also, the *quest*-
or demand, lies upon the *third* number.

yards qrs.
— 4 — 3
5 first number

3)20

f. $6\frac{2}{3}$

Ans. 6 yards, and 2 remains, which I place a-top of the 3, thus $\frac{2}{3}$, and it is 2-thirds: so is the answer $6\frac{2}{3}$ yards.

quest. 2. If, when wheat is 3s. 6d. a bushel, the penny loaf weighs 4 lb. 2 oz. what ought it to be when the wheat is 5s. a bushel? *Ans.* 2 lb. 11 oz.

s. d.	lb. oz.	s.
If 3 - 6	4 - 2	5
12	12 oz. 1 lb. troy	12
42 pence	50	60
	42	
	100	
	200	
	610)21010	
	12)35 oz.	
	2 lb. 11 oz.	<i>Ans.</i>

Quest.

Quest. 3. A regiment of soldiers, consisting of men, are to have new coats, each containing 3 yds. 2 qrs. of cloth, that is 6 qrs. wide, and they are lined with shalloon that is yard wide; how much will it take for their cloaths, and how much shalloon will line them? *Ans.* 5250 yards.

First, see how much is in their cloaths, thus:

$$\begin{array}{r} \text{Yds. qrs.} \\ 1 \text{ coat is } 3 - 2 \\ \underline{4} \end{array}$$

that is 14 qrs.

1000

14000 qrs. in all

$$\begin{array}{r} \text{qrs.} \quad \text{qrs.} \quad \text{Yard} \\ \text{Then, if } 6 \text{ --- } 14000 \text{ --- } 1 \text{ shalloon} \\ \quad \quad \quad 6 \quad \quad \quad 4 \end{array}$$

$$\begin{array}{r} \text{Third-number } 4 \text{) } 84000 \quad \quad \quad 4 \text{ qrs.} \\ \underline{\quad \quad \quad} \end{array}$$

$$4 \text{) } 21000 \text{ qrs.}$$

5250 yards. *Ans.*

The RULE OF THREE INVERSE.

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Quest. 4. An acre of land contains (according to the
 40 rods in length, and 4 in breadth, what must
 be length to make an acre, when the breadth is 15
 ?

<i>Breadth</i>	<i>Length</i>	<i>Breadth</i>
If 4	40	15
	4	
15	160	(10 rods
	15	
	10	
	5½	yards 1 rod
		<i>Rods Yds. Feet</i>
15	55	(3 yards. <i>Ans.</i> 10 - 3 - 2
	45	
	10	
	3	feet
15	30	(2 feet
	30	
	0	

Quest. 5. If £. 100 in 12 months gain £. 4 interest,
 principal will gain as much in 8 months? *Ans.*

<i>Months</i>	<i>£.</i>	<i>Months</i>
If 12	100	8
	12	
	8	
	1200	
	£. 150	<i>Ans.</i>

Quest.

Quest. 6. An army of soldiers, consisting of $\frac{1}{2}$ a town, having with them provisions for 3 months; but as they cannot take the town in time, the General is willing to make the same provisions last five months, allowing each man the daily provision: the question is, how many men there be, that the provision may last them five months.

Months	Men	Months	Men
* If 3	800	5	800 for 3 months
	3		480 for 5 months
	5) 2400		320 discharged
	480		

The last QUESTION varied.

* *Note,* This question is set in words on purpose *Tyro*, to puzzle you; but you must observe, that in such sort of sums, the circumstance of what is done is never to be minded; for, suppose I should say thus

If 800 men in three months eat 10000 penny loaf, how many will eat the same in five months, at the same rate? *Ans.* 480 men as before; so that the circumstance of what they do is nothing, let it be eating, drinking, or any other thing.

Tyro. I understand you, sir, and thank you for your observation; and I think you have given me sufficient examples.

Philo. All other questions are done after the same manner. If you are perfect in these, you will easily do the following ones.

QUESTIONS to exercise this RULE.

Quest. 7. If when wheat is sold for 3s. 9d. a bushel, the sixpenny loaf weighs 5 lb. 10 oz. how much

weigh when it is half a crown a bushel? *Ans.* 9 oz.

quest. 8. How many yards of matting, that is 2 wide, will cover a room 20 feet long, and 18 feet 2. *Ans.* 180 feet, or 60 yards.

quest. 9. How many yards of paper, three quarters 2, will be sufficient to hang a room that is 24 yards d, and 4 yards high? *Ans.* 128 yards.

Tyro. These questions are sufficient, sir.

philo. Then we will proceed to *Section 2.*

SECTION II.

PLURAL PROPORTION;

CALLED

The DOUBLE RULE OF THREE DIRECT.

WHAT is *Plural Proportion*, or the *Double Rule of Three Direct*?

philo. *Plural Proportion* is when five numbers are n to find a sixth, which sixth number, or answer, und by two statings of the *Single Rule of Three* et.

Tyro. This is very difficult, I imagine; for how am know how to make the first stating?

DEFINITION and RULE.

philo. In every regular sum in the *Double Rule of* e *Direct*, the first three numbers always shew the ition, or supposition; and the other two, that is, wo last, shew the demand. The answer of the ion will always be the same as the *middle* num- and the *first* and *third* must be of the same e in both statings, like as in the *Single Rule of* e *Direct*.

Tyro.

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Tyro. Cannot these questions be done at one stating?

Philo. Yes, by a rule generally called the Rule of Three of Five Numbers, and that with a great deal more ease, as you shall see by and by; but we will give an example here.

Quest. If £.100 in 12 months gain £.4 interest, what will £.175 gain in 9 months. *Ans.* £.5 - 5s.

$$\begin{array}{r}
 \text{First, If } 100 \text{ gain } 4 \text{ what will } 175 \\
 \begin{array}{r}
 \text{£.} \quad \text{£.} \quad \text{£.} \\
 100 \quad 4 \quad 175 \\
 \hline
 100 \quad 7 \quad 100 \\
 \hline
 \text{£.} 7
 \end{array}
 \end{array}$$

Here I find £.175 gains £.7 in twelve months, therefore I say in the second stating,

$$\begin{array}{r}
 \text{Months} \quad \text{£.} \quad \text{Months} \\
 \text{If } 12 \text{ --- } 7 \text{ --- } 9 \\
 \quad \quad \quad 9 \\
 \quad \quad \quad \hline
 12 \quad 63 \\
 \quad \quad \quad \hline
 \text{£.} 5 \text{ --- } 3 \text{ remains} \\
 \quad \quad \quad 20 \\
 \quad \quad \quad \hline
 \text{Same divisor} \quad 12 \quad 60 \\
 \quad \quad \quad \hline
 \quad \quad \quad 5 \text{ shillings}
 \end{array}$$

Or thus.

You may begin with the second, third, and fourth numbers.

$$\begin{array}{r}
 \text{Months} \quad \text{£.} \quad \text{Months.} \\
 \text{First, If } 12 \text{ --- } 4 \text{ --- } 9 \quad \text{Ans. £.} 3
 \end{array}$$

Then, If $\begin{matrix} \text{£.} & & \text{£.} & & \text{£.} \\ 100 & \text{---} & 3 & \text{---} & 175 \end{matrix}$ Ans. $\text{£.} 5 - 5$.

Note, See the proof of this question in the first example of the next *Rule Inverse*.

Quest 2. If 600 seamen in one week eat 1500 lb. of bread, how many pounds will serve 120 seamen 12 weeks? Ans. 3600 lb. See *Example 2*, in the next rule.

All questions in this rule being done after the same manner, I shall leave other examples for the *Rule of Five Numbers*, by which they are done with ease; and shall only give you a notion of stating questions here and in the *Double Rule of Three Inverse*.

RULE to state, or know the Nature of a Question, and to know the Certainty of your three first Numbers, and the Name of the Answer.

You know there are five numbers given to find out the answer; mind then you carefully observe what name the answer is to be in: this being done, place your numbers in two rows, viz. three in the first row and two under them, leaving a blank space under that number which is of the same name as your answer is to be in; so will you see both here, and in the next rule, the order of both these *Double Rules of Three Direct and Inverse*.

Take the first question, where your answer is to be in pounds, and rank the numbers thus, as the question runs.

If $\text{£.} 100 - 12 \text{ months} - \text{£.} 4$.

$\text{£.} 175 - 9 \text{ months} \quad *$

Here you see the blank or vacant place is money: therefore, money must be your middle number of the three, and money will be the answer.

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Note. The blank must fall under the supposition which here is £. 4.

Second QUESTION.

If 600 seamen—1 week—1500 lb.

120 seamen—12 weeks *

* This third number in the second row has a blank place under, you see, according to the order of the sum; therefore 1500 lb. must be your *second* number, the first stating, and your answer will also be lbs.

Tyro. I humbly thank you, sir.

Philo. There are many other ways of varying the rule; but this is sufficient for your purpose at present. See the *Rule of Five Numbers*.

SECTION III.

The DOUBLE RULE OF THREE INVERSE; called PLURAL PROPORTION.

Tyro. WHAT is the difference between this and the last?

Philo. A great deal: for though here be five numbers given to find a *sixth*, yet you will find it more difficult to state the question here than in the other; but observe the following direction.

R U L E.

1. As in the *Double Rule Direct*, so also here three numbers in the supposition, and two in the demand.

2. Place the numbers in order, as in the last, viz. three in one row and two in the next, le

The DOUBLE RULE OF THREE INVERSE. 171

that number with a blank place under, which will be the same name of your answer, observing well where the supposition lies.

3. This being done, you state the question with three numbers first, as before; which is wrought according to the *Single Rule of Three Inverse*.

☞ If your first stating be *direct*, the second must be *inverse*; for both are never *direct*, nor never *inverse*.

Quest. 1. If £. 100 principal * in 12 months gain 4 interest, what principal will gain 5 guineas (*viz.* 5 - 5s.) in 9 months? *Ans* £. 175 principal.

* *Note,* The word *principal* signifies the money put to interest.

Here you see there are three numbers in the supposition, *viz.* £. 100, 12 months, and £. 4; and two in the demand, *viz.* £. 5 - 5s. and 9 months.

Now observe, that one of these two in the demand must be the last number in the *first stating*, and the other the last in the *second stating*, and one must be always *inverse*.

Months £. Months
First, *Inverse.* If 12—100—9

Multiply £. 100 by 12, and divide by 9, you have 133 $\frac{1}{3}$, *viz.* £. 133 - 6 - 8. Then,

£. £. s. d. £. s.
Secondly, *Direct.* If 4—133 - 6 - 8—5 - 5

Here I reduce the *first* and *last* numbers into shillings, the *second* into pence, and by the *Rule Direct*, I find the answer £. 175, the principal required.

2. Or thus:

1. *Direct.* If $\begin{array}{ccccc} \text{£.} & \text{£.} & \text{£. s.} \\ 4 & 100 & 5 - 5 \end{array}$ *Ans.* £. 131 - 5s.
 $\begin{array}{ccccc} & \text{Months} & \text{£.} & \text{s.} & \text{Months} \end{array}$

2. *Inverse.* If $12 - 131 - 5 - 9.$ *Ans.* £. 175,
 before.

Quest. 2. If 600 seamen in 1 week eat 1500 lb. of beef, how long will 3600 lb. serve 120 men at the same rate?

$\begin{array}{ccccc} & \text{Men} & \text{Week} & \text{lbs.} \\ \text{Here, If } 600 & 1 & 1500 \\ 3600 \text{ lb.} & * & 120 \text{ men} \end{array}$

First stating, *Inverse.* If $600 - 1 - 120.$ *Ans.*
 weeks.

Secondly, *Direct.* If $1500 - 5 - 3600$ *Ans.*
 weeks.

* See the second question of the last rule.

Tyro. I understand it a little, but cannot say I am perfect in it.

Philo. It is of no great signification at present, you will see more of it in the next rule.

SECTION IV.

The RULE OF THREE composed FIVE NUMBERS.

Tyro. **W**HAT difference is there between this and the other two preceding rules?

Philo. Only this, that all questions in the *Double Rule of Three Direct* and *Inverse*, are performed at one stating.

Tyro. So you said before, I remember; please therefore to give me a rule to work it by; for, I think, it is much better than the trouble of two statings.

Philo. It is so in general, but not always; however, it is easier and better for the learner upon the whole; for here, as well as in the two former rules, the difficulty lies to know whether the question be *direct*, or *inverse*. This being known, the manner of stating the question will be easy, and the work as easy.

A standing Rule to state Questions.

1. As in the *Single Rule of Three Direct*, so also here, the *middle number* governs the question, and the answer will be of the same name.

2. Having noted which the *middle number* is, the other four are easily known; for the *two* that are in the supposition always are the *two* before the *middle number*; the other two are the demand under the *third and fourth numbers*.

Lastly, Always observe, that your *first number* be of the same name as your *fourth*; and your *second number* be the same as your *fifth*. This being done,

1. *The Rule for all direct Questions is this:*

Multiply your *first number* by your *second*, for a *divisor*; then multiply your *third, fourth, and fifth* together, and divide their product by the product of the *divisor* and *second*, and the quotient will be of the same name as the *middle number*, be what it will, and is the answer.

174 *The RULE OF THREE of Five Numbers.*

EXAMPLE. *The first Question of the two last Rules.*

Quest. 1. If £. 100 in 12 months gain £. 4 interest, what will £. 175 gain in 9 months? *Ans.* £. 5 - 5s.

£. Months	£.	£. Months.
If 100—12 gain 4	—	175—9
12		9
1200		1575
		4
		12 00)63 00
		£. 5 - 3 remains
		20
		12)60
		5 shillings

Ans. £. 5 - 5s. See the first example of the last rule.

Quest. 2. If 600 seamen, in 1 week, eat 1500 lb. beef, how many lb. will serve 120 seamen 12 weeks?

Men Week	lb.	Men Weeks
If 600—1	—	1500—120
1		120
600		180000
		12
		6 00)21600 00
		3600 lbs. <i>Ans.</i>

See Ex. 2, in the *Double Rule Direct.*

Another Rule for the more easy stating a Question.

Note. If you observe the two foregoing examples, they are stated just as the numbers follow one another in the words of the question; but it very often happens that the numbers in the sum lie contrary to the order they ought to be placed in; therefore, you should turn the question into other words, and it will be easier. Or,

2. Set all the numbers down as they run in the question, writing the names over each; then consider what your answer is to be in, and place that down beneath for the *middle number*; then the *two* where the demand lies must be your *fourth* and *fifth*, and the other two your *first* and *second* numbers: the *first* must be the same as the *fourth*, and the *second* the same as the *fifth*. See the next question.

Quest. 3. If £.4 be the wages of 8 men 10 days, what will it cost me, or what must be the wages of 32 men 12 days? Ans. £.19 - 4s.

Here, as before directed, I place the numbers after one another, thus:

£. Men Days Men Days
If 4—8—10—32—12. Wrong stated!

Now, here the *middle number* is days, but should be money, because the question says, what will it cost for 32 men, 12 days; therefore money must be my *middle number*, and then, *Tyro*, the true *stating* will appear easy, as follows:

Men Days £. Men Days
thus *, 8—10—4—32—12. Ans. £.19 - 4s.

Now here, *Tyro*, the *stating* runs like the words of the sum, as follows, which is very natural.

* If 8 men in 10 days cost me £.4 for wages, what will 32 men cost me in 12 days, at that rate? Ans. as before.

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Quest. 4. (A proof of the first question in the last rule.) A person put out 175 pounds to receive interest, and when it had continued nine months, he received principal and interest together £. 180 - 5. I demand at what rate *per cent. per annum* he put his money out at? *Ans.* £. 4 *per cent. per annum.*

Here the words of the question are like the *stating*. You must observe then to proceed as under, in all questions of this nature.

Rule Take the principal from the interest and principal together, the remainder will be the interest for the time it continued, and then the stating will follow:

Thus, Principal and interest	—	£. 180 - 5s.
Principal put out	—	175 -
Interest for 9 months	—	£. 5 - 5s.

£.	Months	£.	s.	£.	M
If 175	— 9	gain 5	- 5	interest,	what will 100 in 12

Here, according to the rule, bring your *middle number* into shillings; then multiply it by your *fourth* and *fifth* for a *dividend*; this done, multiply your *first* and *second* together for a *divisor*, and the quotient will be 80 shillings, or £. 4, the rate he put the money out at.

Tyro. I heartily thank you; and I find by this I shall be able to answer Mr. Coker's last question in the *Double Rule of Three Direct*, which, before always appeared very difficult to me.

Philo. You say very right, *Tyro*, for the question done after the very same manner; therefore shall give you only two or three examples in the *Double Rule of Three Inverse*, and leave you to practise the other examples.

QUESTION

QUESTIONS Inverse.

2. The rule for *Questions Inverse* is as follows: the numbers belonging to the supposition must be your *first, second, and third* numbers, and the demand is your *fourth and fifth* number. These being observed, the rule is, multiply your *second, third, and fourth* together for a *dividend*, and your *first and fifth* together for a *divisor*, and the *quotient* is the same answer as your *middle number*.

Quest. 1. (A proof to the very last question, and of question 1 in the last rule.) If £ 100 in 12 months gain 4 interest, what principal will 5 guineas gain in 9 months?

Here, as the principal is to be the answer, so the principal (*viz.* £. 100) must be your *middle number*.

£.	Mths.	£.	£.	s.	Months	£.
If 4	— 12	be 100	principal,	what 5	— 5	be 9
						<i>Ans.</i> 175.

I state the question according to the first standing rule, making my *first* and *fourth*, and my *second* and *fifth* alike; this done, I bring my *first* number into shillings, (*viz.* 80) because the *fourth* must be brought into shillings: this being done, I multiply the *second, third, and fourth* together, for a *dividend*, and the *first and fifth* together for a *divisor*, and the *quotient* is £. 175, the principal required.

Quest. 2. If 600 seamen in 1 week eat 1500 lb. of beef, how long will 3600 lb. last 120 seamen?

lb.	Men	Week	lb.	Men
If 1500	— 600	— 1	— 3600	— 120

Here I multiply the *second, third, and fourth* together for a *dividend*, which is 2160000, and my *first and fifth* for a *divisor*, which is 180000, and the *quotient* is 12 weeks.

Quest. 3. If it cost £.240 for 60 servants in 8 weeks, how long will £.140 serve 8 servants? *Ans.* 35 weeks.

Tyro. Sir, these examples are sufficient, I think.

Philo. If you are well grounded in the nature of the rule, that is enough; for you may set yourself sums at any time by way of exercise. And now, *Tyro*, I will give you a notion of *Tare* and *Tret*.

SECTION V.

T A R E and T R E T.

Tyro. **W**HAT do you mean by *Tare* and *Tret*?

Philo. *Tare* and *Tret* are allowances made in buying and selling commodities that are liable to loss or waste.

Tyro. What are the general names belonging to this rule?

Philo. They are 6, viz. *gross*, *tare*, *tret*, *futtle*, *cloff*, and *neat*.

Tyro. Please to explain these in their order.

Philo. *First*, *Gross* signifies the whole weight of any commodity, or parcel of goods, bag, or box, or cask, and all included: thus, suppose I had a box of tea, bag of spice, or cask of oil, which weighs 120 lb. I say it weighs 120 lb. *gross*.

Secondly, *Tare* is an allowance made for the weight of the box, bag, or cask, and what hangs about it, and is to be taken out of the *gross*, and then the remainder is called the *neat*, or *clear weight* of the commodity: thus, suppose the foregoing box, bag, or cask, after the goods are out, should weigh 16 lb. then I say

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Thus,

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104

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Sixthly
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I say, it has 16 *lb.* *tare*, which I subtract out of the *gross* 120 *lb.* and there remains 104 *lb.* *neat*, thus:

Bought a box of tea, *Gross* 120 *lb.*
Tare 16 *lb.*

Neat 104 *lb.*

Thirdly, *Tret* is an allowance of 4 *lb.* for every 104; that is 1 *lb.* is allowed for every 26 *lb.* for such commodities as are liable to waste, moth, dust, &c.

Note, When there is *tret* in the sum, then after the *tare* is taken from the *gross*, the remainder is called *suttle*, instead of *neat*.

Note 2. To find the *tret* pounds, divide the *suttle* pounds by 26, and the *quotient* is the *tret*, which subtracted from the *suttle* gives the *neat*.

Thus, A box of tea, *Gross* 120 *lb.*
Tare 16

26) 104 (4 *tret*.

104

—

0

Suttle 104 *lb.* which divide by 26
Tret 4 *lb.*

—
Neat 100 *lb.*

Fourthly, *Suttle* pounds are so called (as was said before) when there is *tret* in the sum; for when there is no *tret* there is no *suttle*, and the *tret* pounds are always subtracted out of them, as in the above.

Fifthly, *Cloff* is an allowance of 2 *lb.* for every 3 *Cwt.* for the turn of the scale, and for small draughts; but this is allowed only in some particular things and cases.

Sixthly, *Neat* weight is the clear weight of the commodity after all the allowances are made.

Tyro. Sir, I understand you very well; but if I remember, you told me (in *page 142*) you would shew me a shorter method to reduce *cwts.* and *qrs.* into *lbs.* than you did in *page 104.*

Philo. I did so, and will now shew you all the ways, and leave you to take your choice.

To reduce Hundreds and Quarters into Pounds.

EXAMPLE I.

C. qrs. lbs.

Reduce 17 - 3 - 15 into *lbs.*

First method. *C. qrs. lbs.*
17 - 3 - 15
4

Common way 71
28

573

143

Ans. 2003 *lbs.*

C. qrs. lbs.

Second method. 17 - 3 - 15*

17

17

17

Add 99 for the 3 *qrs.* 15 *lb.*

Ans. 2003 *lb.* See p. 104.

Note. In this method you must always set the *C.* down four times, *viz.* twice under one another, and the other two, each one place more towards the left hand; then count how many pounds there are in the odd *qrs.* and *lbs.* (which are here 99) and always place them under the *units, tens,* as above, and adding them together, you have the *answer.*

Note

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Note, This method is shorter than the *first*, and more particular when there are many figures in the given hundreds.

Third method. Multiply by 112, and take in the odd pounds.

$$\begin{array}{rcl} C. & qrs. & lb. \\ 17 & - & 3 & - & 15 \text{ is } 17 \text{ and } 99 \\ & & & & 112^* \end{array}$$

Ans. 2003

Here I multiply by 112 in one line, thus: I say, times 7 is 84, and 9 odd lbs. make 93, 3 and I carry 9; then 12 times 1 is 12, and 9 I carried is 21, the other 9 I take in also is 30, that is 6, and I carry 3. *Lastly*, I say, once 17 is 17, and 3 I carried 20, which I place by the side of the other two figures, it is done.

This is also shorter than the *first method*, but somewhat more difficult till you learn to multiply by 112 well, as you will see in *Ex. 2*.

Fourth method. The best way (in my opinion) of this last method, which is this: multiply the *C*'s by only, always remembering to place the two first figures out towards the right-hand; then set the odd pounds under, and add them together as they stand, it is done.

$$\begin{array}{rcl} C. & qrs. & lbs. \\ \text{Thus, } 17 & - & 3 & - & 15 \text{ is } 17 & - & 99 \text{ +} \\ & & & & 204 \\ & & & & 99 \end{array}$$

Ans. 2003 lbs. as before.

There is no occasion to set down the 12 under the when you multiply; for it is easy to remember you multiply by 12. I say then 12 times 7 is 84, (which

See the note at the top of the next page.

4 I place 2 figures towards the right hand of the that is 4 and I go 8; then 12 times 1 is 12, and 8 is which I place to the left of the 4, so that the 2 is under the 7; then I set down the 99 odd pounds, and add them together, they make 2003 *lb.*

† *Note*, But you may do it yet shorter by use, as full as easy, and that is by taking in the 99 as multiply, which will save the trouble of adding it afterwards.

$$\begin{array}{r}
 C. \quad lb. \\
 \text{Thus, } 17 - 99 \\
 \quad \quad 303 \\
 \hline
 \quad \quad 2003
 \end{array}$$

Here I say, 12 times 7 is 84, and 9 is 93, 3 and carry 9; 12 times 1 is 12, and 9 I carry is 21, and last 9 belonging to the 99 makes 30.

Now, the reason of this will appear plain, if consider that 17 *C.* is 1700 *lb.* and 17 times 12 therefore I only set down 1700 *lb.* and add 12 times 17 to it.

$$\begin{array}{r}
 \text{Thus, } 17 \text{ } C. \text{ is } \text{---} 1700 \\
 \text{More, } 12 \text{ times } 17 \text{ is } 204 \text{ add}
 \end{array}$$

Ans. 1904 *lb.* in 17 *C.*

$$\begin{array}{r}
 1700 - 99 \text{ } lb. \\
 \text{So also } \quad 303 \text{ is } 12 \text{ times } 17, \text{ and } 99. \\
 \hline
 2003 \text{ as before.}
 \end{array}$$

✍ *Note*, Or you may do it shorter yet, only putting or joining the odd *lbs.* by the side of the (making a dot between them for distinction sake) and multiply the *C.*'s only by 12, placing 2 figures out from them, as before directed, and you have the answer which will save the trouble of taking in.

Thus, 17 - 99 I set down 17.99

$$\begin{array}{r} 17.99 \\ 204 \\ \hline \end{array}$$

Ans. 2003 lb.

Here I multiply 17 only by 12, always placing the first figures to the right-hand : then add the two together.

EXAMPLE 2.

47 C. 1 qr. 26 lb. neat, how many lb.

C. qr. lb. C. lb.

47 - 1 - 26 is 47.54 multiply 47 only by 12.

multiplied by 12 is
$$\begin{array}{r} 47.54 \\ 564 \\ \hline \end{array}$$

5318 *Ans.*

EXAMPLE 3.

86 C. 1 qr. 17 lb. neat, how many lbs.

C. qr. lb. C. lb.

86 - 1 - 17 is 86.45

multiplied by 12 is
$$\begin{array}{r} 86.45 \\ 1032 \\ \hline \end{array}$$

9677 *Ans.*

I join the odd 45 lb. close to the 86 C. and then multiply 86 only by 12, placing two figures out as before.

ro. Very well, sir, I understand what you have said to me; but suppose the odd pounds amount to three, may I join them to the C's then?

ilo. No, only two of them; and place the other one (which will always be 1) under the units place of the C's, as in the following example.

Ex-

EXAMPLE 4.

Reduce 89 C. 3 qrs. 21 lb. neat, into lbs.

That is, 89 C. and 105 lb. Thus 89
 3 qrs. 21 lb. is — 105 lb.
 89 multiplied by 12 is — 1068

Ans. 10073 lb.

Here 3 qrs. 21 lbs. make 105 lbs. and because there are three figures, I place the 1 under the 9, and thus all such examples.

Now, *Tyro*, I will prove this last example by other three methods, and leave you to take your choice as practice or fancy direct.

C. qrs. lb.	C. lb.	C.
89 - 3 - 21	Secondly, 89 - 105	Thirdly, 89 -
4	89	112
<hr/>	89	<hr/>
359	89	10073 lb.
28	105	
<hr/>	<hr/>	
2873	10073 lb.	
720		
<hr/>		

10073 lb. common.

* This third method is certainly very short, if you can multiply well by 112: for observe, I say 12 times 9 is 108, and 5 (the odd lbs.) is 113; 3, I carry 11; then 12 times 8 is 96, and 11 is 107, and 10 lb. is 117, that is, 7 and I carry 11; lastly, I say 89 is 89, and 11 I carried is 100.

Tyro. Sir, I heartily thank you; I see now how it is done.

Philo. Then I will give you one example at last.

EXAMPLE 5.

Bought 3 hogheads of sugar, as under.

	<i>C.</i>	<i>qrs.</i>	<i>lb.</i>
<i>Gross</i> No. 1	11	2	15
2	15	1	17
3	17	3	14

of all 1 *C.* 1 *qr.* 2 *lbs.* }
 4 *lb.* for every 104. at
 6 shillings a *C. neat*, what
 come they to?

<i>Gross</i>	44	3	18
<i>Tare</i>	1	1	2

<i>Suttle</i>	43	2	16
---------------	----	---	----

<i>Suttle</i>	4888	<i>lbs.</i>
<i>Tret</i>	188	<i>lbs.</i>

<i>Neat</i>	4700	<i>lbs.</i>
-------------	------	-------------

26) 4888 (188 *Tret*

lb.

228

208

208

208

208

then, if 1 *C.* viz. 112 *lb.* cost 21. 16s. what cost
lbs. neat? *Ans.* £. 117 - 10s.

EXAMPLE 6.

Bought 3 chests of tea, each weighing, *gross*, 1 *C.*
 12 *lb.* tare of each 1 *qr.* 5 *lb.* at 4s. 9d. a *lb. neat*,
 do they come to?

There being no *tret*, the *tare* taken out of the *gross*,
 remainder is *neat*. *Ans.* £. 124 - 13 - 9.

ro. I am highly obliged to you, sir; and am now
 for the next rule, if you please.

ilb. The next thing I shall instruct you in is the
 of *Practice*.

DIALOGUE

D I A L O G U E IX.

SECTION I.

Of PRACTICE.

Tyro. **W**HAT do you mean by *Practice*?
Philo. *Practice* is only *Division* of *Measures* and very much resembles *Reduction ascending*; for you divide by as many of the *less* as make one of *greater*. It is a short contracted way of the *Method of Three Direct*, and saves the trouble of *stating questions*, by dividing by the parts contained in the whole.

Tyro. Are not *Vulgar Fractions* often taught by *Practice*?

Philo. They are; but there is no necessity for it if you can read a fraction, that is enough. Thus $\frac{2}{7}$, $\frac{1}{8}$, or $\frac{1}{12}$, are thus read, 2 *sevenths*, 3 *fourths*, 1 *eighth* and 1 *twelfth*.

Tyro. I know this very well, sir.

Philo. Then you may proceed to work this rule as soon as you please; but it will be necessary to learn the following tables by heart.

T
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mplesWhen t
farthi
gs into
e. $\frac{1}{4}$ $\frac{1}{12}$

2|0

TA

TABLE 1.

TABLE 2.

Parts of a Shilling.

Even Parts of a £. Sterling.

Parts of a Shilling.		Shillings	Even Parts of a £. Sterling.	
is	$\frac{1}{2}$ of a shilling	10	is	$\frac{1}{2}$ of a £.
	$\frac{1}{4}$	6-8d.		$\frac{1}{3}$
	$\frac{1}{8}$	5		$\frac{1}{4}$
	$\frac{1}{16}$	4		$\frac{1}{5}$
	$\frac{1}{32}$	3-4d.		$\frac{1}{6}$
	$\frac{1}{64}$	2-6d.		$\frac{1}{8}$
	$\frac{1}{128}$	2		$\frac{1}{10}$
	$\frac{1}{256}$	1-8d.		$\frac{1}{12}$
	$\frac{1}{512}$	1-3d.		$\frac{1}{16}$
	$\frac{1}{1024}$	1		$\frac{1}{20}$

pro. I will learn these tables as fast as I conveniently can.

hilo. Very well; then I shall proceed to give you examples under the following Cases.

CASE 1.

When the sum is given in farthings, that is, 1, 2, 3, 4 farthings per lb. or yard, then only bring these farthings into pence, shillings, and pounds, and the work is as follows.

$\frac{1}{4}$	1753 lb. at $\frac{1}{4}$ a lb.	Here 1753 lbs. at a farthing a lb. is 1753 farthings, therefore I only divide by 4, by 12, and by 20, and have £.1-16-6 $\frac{1}{4}$ Ans.
$\frac{1}{12}$	438 $\frac{1}{4}$	
2 0	3 6-6d.	
	1-16-6 $\frac{1}{4}$ Ans.	

$\frac{1}{2}$ ny	$\frac{1}{2}$	1753 lb. at $\frac{1}{2}$ ny a lb.	Here 1753 is 1753 ha
1d.	$\frac{1}{12}$	876- $\frac{1}{2}$	pence. I divide therefore
1s.	2 0	7 3	2, to bring them into pen
		£3-13-0 $\frac{1}{2}$ Ans.	and by 12 and 20, to br

them into £'s, and the
swer is £.3-13-0 $\frac{1}{2}$, which
just double the first sum,
a farthing a lb and is a pr
to it.

* $\frac{3}{4}$	$\frac{1}{8}$	1753 lb at $\frac{3}{4}$ per lb.	Here I say, 3 farthings
6d.	$\frac{1}{2}$	219- $\frac{3}{4}$	the 1-8th of six-pence, a
1s.	2 0	10 9-6	dividing by 8, I have
		£5-9-6 $\frac{3}{4}$ Ans.	six-pences, and 3 farthi

over; then I divide by 2
cause 6d. is $\frac{1}{2}$ of a shilli
and have 109 shillings, 2
6d. over; then I divide by
to bring them into £'s, a
have £.5-9-6 $\frac{3}{4}$.

Note, If you add the answers of the first and second sum together, you will find they amount to £.5-9-6 $\frac{3}{4}$ which proves the work.

* There is another way to do this: Multiply 1753 by 3 farthings, and the product is farthings; then divide by 4, 12, and 20.

1753 lb. at $\frac{3}{4}$ per lb.

3

5259

1314- $\frac{3}{4}$

10|9-6

 $\pounds 5-9-6\frac{3}{4}$ Ans.

Yards

1753* at 1d. per yd.

14|6-1

 $\pounds 7-6-1$ Ans.

* Note, I make use of the same number, 1753, because you may prove the work of one sum by another, by adding the products of any two together, thus: the product of 3 farthings and 1 farthing will prove the last sum of a penny.

CASE 2.

When the given price is pence and farthings, then take the parts for the pence first, and then

the parts for the farthings.

d.

1 $\frac{1}{2}$ $\frac{1}{4}$

2|0

Ells

1753 at $1\frac{1}{4}$ d. per ell

146-1d.

36-6 $\frac{1}{4}$ *18|2-7 $\frac{1}{4}$ $\pounds 9-2-7\frac{1}{4}$ Ans.

* Here I say, 1 penny is the $\frac{1}{2}$ of a shilling; then I say, 1 farthing is the $\frac{1}{4}$ of a penny, and dividing 146 shillings by 4, I have 36 shillings, and 2 remains, which is two shillings; then I say, the 4th of 2 shillings is 6d. and the 4th of a penny is $\frac{1}{4}$; then I add them both together, which make 182s. 7d. $\frac{1}{4}$, and dividing by 20, I have $\pounds 9-2-7\frac{1}{4}$ for answer.

☞ Always remember that what remains is shillings and pence, and you must also divide such shillings by the same figure, and you

you have the true answer.

d. $1\frac{1}{2}$ $\frac{1}{8}$ *lb.*
1753 at $1\frac{1}{2}d.$ per *lb.*

$2|0$ 21|9- $1\frac{1}{2}$

$\pounds 10-19-1\frac{1}{2}^*$ *Ans.*

* This you see is just double the sum of three farthings.

d. $1\frac{1}{2}$ $\frac{1}{8}$ *lb.*
1753 at $1\frac{3}{4}d.$

$\frac{1}{4}$ $\frac{1}{6}$ 219- $1\frac{1}{2}$
36- $6\frac{1}{4}^*$

$2|0$ 25|5- $7\frac{3}{4}$

$\pounds 12-15-7\frac{3}{4}$

* Note, Here I say, 3 half-pence is the $\frac{1}{8}$ th (as before) then I say, 1 farthing is the $\frac{1}{6}$ th of 3 half-pence, and dividing by 6, I have 36, and 3 remains, which 3 I call 3 shillings; then I say, the 6th part of 3 shillings is 6 pence, and the 6th part of $1\frac{1}{2}$ is $\frac{1}{4}$. Thus you see I take the parts of the remainder as

well as the of the whole.

Pray, observe *ro*, and look we the example; shall now leave to yourself while.

Yards

2d $\frac{1}{6}$ 1753 at $2d.$ per

$2|0$ 29|2- $2d.$

$\pounds 14-12-2$ *Ans.*

2d $\frac{1}{6}$ 1753 *lb.* at $2\frac{1}{4}d.$

$\frac{1}{4}$ $\frac{1}{8}$ 292- $2d.$
36- $6\frac{1}{4}$

$2|0$ 32|8- $8\frac{1}{4}$

$\pounds 16-8-8\frac{1}{4}$ *Ans.*

2d $\frac{1}{6}$ 1753 *oz.* at $2\frac{1}{2}d.$

$\frac{1}{2}$ $\frac{1}{4}$ 292- $2d.$
73- $0\frac{1}{2}$

$2|0$ 36|5- $2\frac{1}{2}$

$\pounds 18-5-2\frac{1}{2}$

2d $\frac{1}{6}$ 1753 *Ells* at

$\frac{1}{2}$ $\frac{1}{4}$ 292- $2d.$
73- $0\frac{1}{2}$
36- $6\frac{1}{4}$

$2|0$ 40|1- $8\frac{3}{4}$

$\pounds 20-1-8\frac{3}{4}$

lb.

1753 at 3d.

43|8-3

 $\pounds 21-18-3$ Ans.

lb.

1753 at 3 $\frac{1}{4}$ d.438-3
36-6 $\frac{1}{4}$ 47|4-9 $\frac{1}{2}$ $\pounds 23-14-9\frac{1}{4}$ Ans.

lb.

1753 at 3 $\frac{1}{2}$ d.438-3d.
73-0 $\frac{1}{2}$ 51|1-3 $\frac{1}{2}$ $\pounds 25-11-3\frac{1}{2}$ Ans.

lb.

1753 at 3 $\frac{3}{4}$ d.438-3d.
109-6 $\frac{3}{4}$ 54|7-9 $\frac{3}{4}$ $\pounds 27-7-9\frac{3}{4}$ Ans.

Dozen

1753 at 4d.

58|4-4

 $\pounds 29-4-4$ Ans.

3d

 $\frac{1}{4}$

1d

 $\frac{1}{3}$ $\frac{1}{4}$

2|0

4d

 $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{8}$

2|0

4d

 $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{4}$

2|0

4d

 $\frac{1}{3}$

1d

 $\frac{1}{4}$

2|0

lb.

1753 at 4 $\frac{1}{4}$ d.438-3
146-1
36-6 $\frac{1}{4}$ 62|0-10 $\frac{1}{4}$ $\pounds 31-0-10\frac{1}{4}$ Ans.

lb.

1753 at 4 $\frac{1}{2}$ d.584-4
73-0 $\frac{1}{2}$ 65|7-4 $\frac{1}{2}$ $\pounds 32-17-4\frac{1}{2}$ Ans.

lb.

1753 at 4 $\frac{3}{4}$ d.584-4d.
73-0 $\frac{3}{4}$
36-6 $\frac{1}{4}$ 69|3-10 $\frac{3}{4}$ $\pounds 34-13-10\frac{3}{4}$ Ans.

lb.

1753 at 5d.

584-4
146-1

73|0-5

 $\pounds 36-10-5$ Ans.

4d.

<i>Ells</i>	
4d $\frac{1}{3}$	1753 at $5\frac{1}{4}d.$
1d $\frac{1}{4}$	584-4
$\frac{1}{4}$	146-1
	36-6 $\frac{1}{4}$
2 0	76 6-11 $\frac{1}{4}$
	<u>£38-6-11$\frac{1}{4}$ Ans.</u>
<i>lb.</i>	
4d $\frac{1}{3}$	1753 at $5\frac{1}{2}d.$
1d $\frac{1}{4}$	584-4
$\frac{1}{2}$	146-1
	73-0 $\frac{1}{2}$
2 0	80 3-5 $\frac{1}{2}$
	<u>£40-3-5$\frac{1}{2}$ Ans.*</u>
* Or you may take the parts thus, viz. 4d. is $\frac{1}{3}$ of a shilling, as before. Or thus,	
<i>lb.</i>	
4d $\frac{1}{3}$	1753 at $5\frac{1}{2}d.$
1 $\frac{1}{2}$ $\frac{1}{8}$	584-4d.
	219-1 $\frac{1}{2}$
2 0	80 3-5 $\frac{1}{2}$
	<u>£40-3-5$\frac{1}{2}$ Ans.</u>
† Here I say, 1 $\frac{1}{2}$ is $\frac{1}{8}$ of a shilling, and take it out of 1753, the top line, which answers the same end as the last work.	

4d $\frac{1}{3}$	1753 <i>lb.</i> at $5\frac{1}{4}d.$
1 $\frac{1}{2}$ $\frac{1}{4}$	584-4d.
	219-1 $\frac{1}{2}$
	36-6 $\frac{1}{4}$
2 0	83 9-11 $\frac{1}{4}$
	<u>£41-19-11$\frac{1}{4}$ Ans.</u>
<i>lb.</i>	
6d $\frac{1}{2}$	1753 at 6d.
2 0	87 6-6
	<u>£43-16-6 Ans.</u>
<i>Yards</i>	
3d $\frac{1}{4}$	1753 at $6\frac{1}{4}d.$
d.	
3* $\frac{1}{4}$	438-3
$\frac{1}{12}$	438-3
	36-6 $\frac{1}{4}$
2 0	91 3-0 $\frac{1}{4}$
	<u>£45-13-0$\frac{1}{4}$ Ans.</u>
* Here I say, pence is the $\frac{1}{12}$ th a shilling in the cond line as well in the first, because then $\frac{1}{4}$ will be $\frac{1}{12}$; whereas, it was to say 6d. the $\frac{1}{2}$ of a shilling then I must say the $\frac{1}{24}$ of 6 pence now 24 is not for to divide by as 12 so by bringing pence into 2 pence	

the work is a great deal more easy and natural.

Busbels

1753 at $6\frac{1}{2}d.$

876-6

73-0 $\frac{1}{2}$

94|9-6 $\frac{1}{2}$

£47-9-6 $\frac{1}{2}$ *Ans.*

oz.

1753 at $6\frac{3}{4}d.$

876-6

109-6 $\frac{3}{4}$

98|6-0 $\frac{3}{4}$

£49-6-0 $\frac{3}{4}$ *Ans.*

1753 *lb.* at $7d.$

876-6d.

146-1d.

102|2-7

£51-2-7 *Ans.*

1753 *gall.* at $7\frac{1}{4}d.$

876-6d.

146-1

36-6 $\frac{1}{4}$

105|9-1 $\frac{1}{4}$

£52-19-1 $\frac{1}{4}$ *Ans.*

6d $\frac{1}{2}$

1 $\frac{1}{2}$ $\frac{1}{4}$

2|0

4d $\frac{1}{3}$

3d $\frac{1}{4}$
 $\frac{3}{4}$

2|0

4d $\frac{1}{3}$

4d $\frac{1}{3}$

2|0

lb.

1753 at $7\frac{1}{2}d.$

876-6

219-1 $\frac{1}{2}$

109|5-7 $\frac{1}{2}$

£54-15-7 $\frac{1}{2}$ *Ans.*

Ells

1753 at $7\frac{3}{4}d.$

584-4

438-3

109-6 $\frac{3}{4}$

113|2-1 $\frac{3}{4}$

£56-12-1 $\frac{3}{4}$ *Ans.*

Or you may say,
6d. is $\frac{1}{2}$, and 1d. is $\frac{1}{6}$, and $\frac{3}{4}$ is the $\frac{1}{8}$ of 6d.

Yards

1753 at 8d.

584-4

584-4

116|8-8

£58-8-8 *Ans.*

Or you may say,
3d. is $\frac{1}{2}$, and 2d. is $\frac{1}{3}$ of 6 pence; or as 8d. is $\frac{2}{3}$ of a shilling, you may multiply by 3, and divide by 2.

		<i>lb.</i>
6d	$\frac{1}{2}$	1753 at $8\frac{1}{4}d.$
2d	$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$	876-6 292-2 36-6 $\frac{1}{4}$
2 0		120 5-2 $\frac{1}{4}$
		£60-5-2 $\frac{1}{4}$ <i>Anf.</i>
		<i>Pair</i>
4d	$\frac{1}{3}$	1753 at $8\frac{1}{2}d.$
4d	$\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{8}$	584-4 584-4 73-0 $\frac{1}{2}$
2 0		124 1-8 $\frac{1}{2}$
		£62-1-8 $\frac{1}{2}$ <i>Anf.</i>
		<i>Ells</i>
4d	$\frac{1}{3}$	1753 at $8\frac{3}{4}d.$
4d	$\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{4}$	584-4 584-4 73-0 $\frac{1}{2}$ 36-6 $\frac{1}{4}$
2 0		127 8-2 $\frac{3}{4}$
		£63-18-2 $\frac{3}{4}$ <i>Anf.</i>
6d	$\frac{1}{2}$	1753 <i>lb.</i> at 9d.
3d	$\frac{1}{2}$	876-6 438-3
2 0		131 4-9
		£65-14-9 <i>Anf.</i>

		<i>Yards</i>
6d	$\frac{1}{2}$	1753 at $9\frac{1}{4}d.$
3d	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{2}$	876-6 438-3 36-6 $\frac{1}{4}$
2 0		135 1-3 $\frac{1}{4}$
		£67-11-3 $\frac{1}{4}$ <i>Anf.</i>
		<i>Ells</i>
6d	$\frac{1}{2}$	1753 at $9\frac{1}{2}d.$
3d	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{6}$	876-6 438-3 73-0 $\frac{1}{2}$
2 0		138 7-9 $\frac{1}{2}$
		£69-7-9 $\frac{1}{2}$ <i>Anf.</i>
		<i>lb.</i>
6d	$\frac{1}{2}$	1753 at $9\frac{3}{4}d.$
3d	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$	876-6 438-3 109-6 $\frac{3}{4}$
2 0		142 4-3 $\frac{3}{4}$
		£71-4-3 $\frac{3}{4}$ <i>Anf.</i>
		<i>Ells</i>
6d	$\frac{1}{2}$	1753 at 10d.
4d	$\frac{1}{3}$	876-6 584-4
2 0		146 0-10
		£73-0-10 <i>Anf.</i>

	<i>oz.</i>
175	175
876	876
438	438
146	146
36	36
149	149
£74-	£74-
1753	1753
876	876
438	438
219	219
153 3	153 3
£76-	£76-
1753	1753
876-	876-
438-	438-
219-	219-
36-	36-
157 0-	157 0-
£78-	£78-
1753 0	1753 0
876-	876-
584-	584-
146-	146-
160 6-	160 6-
£80-6-	£80-6-

oz.	6d	Ells
1753 at $10\frac{1}{4}d.$	$\frac{1}{2}$	1753 at $11\frac{1}{4}d.$
<u>876-6</u>	$4d$	<u>876-6</u>
438-3	$1d$	584-4
146-1	$\frac{1}{4}$	146-1
<u>36-6$\frac{1}{4}$</u>	$\frac{1}{4}$	<u>36-6$\frac{1}{4}$</u>
149 7-4 $\frac{1}{4}$	2 0	164 3-5 $\frac{1}{4}$
<u>£74-17-4$\frac{1}{4}$ Ans.</u>		<u>£82-3-5$\frac{1}{4}$ Ans.</u>
1753 lb. at $10\frac{1}{2}d.$	$6d$	lb.
<u>876-6</u>	$\frac{1}{2}$	1753 at $11\frac{1}{2}d.$
438-3	$4d$	<u>876-6</u>
<u>219-1$\frac{1}{2}$</u>	$1\frac{1}{2}$	584-4
153 3-10 $\frac{1}{2}$	$\frac{1}{4}$	<u>219-1$\frac{1}{2}$</u>
<u>£76-13-10$\frac{1}{2}$ Ans.</u>	2 0	167 9-11 $\frac{1}{2}$
1753 oz. at $10\frac{3}{4}d.$	$6d$	<u>£83-19-11$\frac{1}{2}$ Ans.</u>
<u>876-6</u>	$\frac{1}{2}$	lb.
438-3	$4d$	1753 at $11\frac{3}{4}d.$
<u>219-1$\frac{1}{2}$</u>	$1\frac{1}{2}$	<u>876-6</u>
<u>36-6$\frac{3}{4}$</u>	$\frac{1}{4}$	584-4
157 0-4 $\frac{3}{4}$	$\frac{1}{6}$	<u>219-1$\frac{1}{2}$</u>
<u>£78-10-4$\frac{3}{4}$ Ans.</u>	2 0	<u>36-6$\frac{3}{4}$</u>
1753 oz. at 11d.	15	171 6-5 $\frac{3}{4}$
<u>876-6</u>	2 0	<u>£85-16-5$\frac{3}{4}$ Ans.</u>
584-4		lb.
<u>146-1</u>		175 3 at 12d.
160 6-11		<u>£87-13 Ans.</u>
<u>£80-6-11 Ans.</u>		

		<i>lb.</i>
6d	$\frac{1}{2}$	1753 at $8\frac{1}{4}d.$
		<hr/>
2d	$\frac{1}{3}$	876-6
$\frac{1}{4}$	$\frac{1}{8}$	292-2
		<hr/>
		36-6 $\frac{1}{4}$
		<hr/>
2 0		120 5-2 $\frac{1}{4}$
		<hr/>
		£60-5-2 $\frac{1}{4}$ Ans.
		<hr/>
		<i>Pair</i>
4d	$\frac{1}{3}$	1753 at $8\frac{1}{2}d.$
		<hr/>
4d	$\frac{1}{3}$	584-4
$\frac{1}{2}$	$\frac{1}{8}$	584-4
		<hr/>
		73-0 $\frac{1}{2}$
		<hr/>
2 0		124 1-8 $\frac{1}{2}$
		<hr/>
		£62-1-8 $\frac{1}{2}$ Ans.
		<hr/>
		<i>Ells</i>
4d	$\frac{1}{3}$	1753 at $8\frac{3}{4}d.$
		<hr/>
4d	$\frac{1}{3}$	584-4
$\frac{1}{2}$	$\frac{1}{8}$	584-4
$\frac{1}{4}$	$\frac{1}{2}$	73-0 $\frac{1}{2}$
		<hr/>
		36-6 $\frac{1}{4}$
		<hr/>
2 0		127 8-2 $\frac{3}{4}$
		<hr/>
		£63-18-2 $\frac{3}{4}$ Ans.
		<hr/>
6d	$\frac{1}{2}$	1753 <i>lb.</i> at 9d.
		<hr/>
3d	$\frac{1}{2}$	876-6
		<hr/>
		438-3
		<hr/>
2 0		131 4-9
		<hr/>
		£65-14-9 Ans.

		<i>Yards</i>
6d	$\frac{1}{2}$	1753 at 9 $\frac{1}{4}d.$
		<hr/>
3d	$\frac{1}{2}$	876-6
$\frac{1}{4}$	$\frac{1}{12}$	438-3
		<hr/>
		36-6 $\frac{1}{4}$
		<hr/>
2 0		135 1-3 $\frac{1}{4}$
		<hr/>
		£67-11-3 $\frac{1}{4}$ Ans.
		<hr/>
		<i>Ells</i>
6d	$\frac{1}{2}$	1753 at 9 $\frac{1}{2}d.$
		<hr/>
3d	$\frac{1}{2}$	876-6
$\frac{1}{2}$	$\frac{1}{6}$	438-3
		<hr/>
		73-0 $\frac{1}{2}$
		<hr/>
2 0		138 7-9 $\frac{1}{2}$
		<hr/>
		£69-7-9 $\frac{1}{2}$ Ans.
		<hr/>
		<i>lb.</i>
6d	$\frac{1}{2}$	1753 at 9 $\frac{3}{4}d.$
		<hr/>
3d	$\frac{1}{2}$	876-6
$\frac{3}{4}$	$\frac{1}{4}$	438-3
		<hr/>
		109-6 $\frac{3}{4}$
		<hr/>
2 0		142 4-3 $\frac{3}{4}$
		<hr/>
		£71-4-3 $\frac{3}{4}$ Ans.
		<hr/>
		<i>Ells</i>
6d	$\frac{1}{2}$	1753 at 10d.
		<hr/>
4d	$\frac{1}{3}$	8-6-6
		<hr/>
		584-4
		<hr/>
2 0		146 0-10
		<hr/>
		£73-0-10 Ans.

oz.	6d	$\frac{1}{2}$	Ells
1753 at $10\frac{1}{4}d$.			1753 at $11\frac{1}{4}d$.
876-6	4d	$\frac{1}{3}$	876-6
438-3	1d	$\frac{1}{4}$	584-4
146-1	$\frac{1}{4}$	$\frac{1}{4}$	146-1
36-6 $\frac{1}{4}$			36-6 $\frac{1}{4}$
149 7-4 $\frac{1}{4}$			2 0 164 3-5 $\frac{1}{4}$
$\pounds 74-17-4\frac{1}{4}$ Ans.			$\pounds 82-3-5\frac{1}{4}$ Ans.
1753 lb. at $10\frac{1}{2}d$.	6d	$\frac{1}{2}$	lb.
876-6			1753 at $11\frac{1}{2}d$.
438-3	4d	$\frac{1}{3}$	876-6
219-1 $\frac{1}{2}$	1 $\frac{1}{2}$	$\frac{1}{4}$	584-4
153 3-10 $\frac{1}{2}$			219-1 $\frac{1}{2}$
$\pounds 76-13-10\frac{1}{2}$ Ans.			2 0 167 9-11 $\frac{1}{2}$
1753 oz. at $10\frac{3}{4}d$.	6d	$\frac{1}{2}$	$\pounds 83-19-11\frac{1}{2}$ Ans.
876-6			lb.
438-3	4d	$\frac{1}{3}$	1753 at $11\frac{3}{4}d$.
219-1 $\frac{1}{2}$	1 $\frac{1}{2}$	$\frac{1}{4}$	876-6
36-6 $\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	584-4
157 0-4 $\frac{3}{4}$			219-1 $\frac{1}{2}$
$\pounds 78-10-4\frac{3}{4}$ Ans.			36-6 $\frac{3}{4}$
1753 oz. at 11d.			2 0 171 6-5 $\frac{3}{4}$
876-6			$\pounds 85-16-5\frac{3}{4}$ Ans.
584-4	1s	2 0	lb.
146-1			175 3 at 12d.
160 6-11			$\pounds 87-13$ Ans.
$\pounds 80-6-11$ Ans.			

And thus, *Tyro*, I have shewn you, from one farthing to a shilling, how to take the parts, so as to find any sum for any number of *lbs. yards, &c.* You may by your own practice take the parts in what manner you please, only remember this *standing rule*, That when you take the parts of a shilling, you must divide the top number by such parts; but when you take the parts of any part, you must divide such a part by the number of parts, and not the top line. Do you understand it?

Tyro. I do, sir, very well; but pray, suppose the price exceeds one shilling, how must I do then?

Philo. That you will soon learn by the following CASE.

CASE 3.

When the given price exceeds one shilling, or pence, but is less than two shillings, then let the line, or given number, stand as it is given, without drawing any line under it; for that is the price or value at 1 shilling; then take the parts as before out of the top line, or given number, and add the said parts to the top number together, you have the answer in shillings, pence, and farthings, which divide by 20, as before.

EXAMPLES *above twelve Pence, and under two Shillings*

3d $\frac{1}{4}$	1753 lb. at 15d. 438 - 3d.
2 0	219 1 - 3
	£109-11-3 <i>Ans.</i>
	Here you see that I let 1753 stand without drawing a line, and that is of itself an answer for 12d. a lb. Then I say, as before, 3d.

	is $\frac{1}{4}$, and add these two together must be the price 15d. for 12, and is 15.
6d $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{4}$	1753 ells at 19 876 - 6d. 219 - 1 $\frac{1}{2}$
2 0	284 8-7 $\frac{1}{2}$
	£142-8-7 $\frac{1}{2}$

d $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{6}$
2|0 8
7
2
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EXAMPLES of other Numbers.

$\frac{1}{2}$	47 yards, at $21\frac{1}{2}d.$	$6d$	$\frac{1}{2}$	*85 ells, at $22\frac{1}{2}d.$
$\frac{1}{2}$	23 - 6d.	$3d$	$\frac{1}{2}$	42 - 6
$\frac{1}{2}$	11 - 9	$1\frac{1}{2}$	$\frac{1}{2}$	21 - 3
$\frac{1}{2}$	1 - $11\frac{1}{2}$			10 - $7\frac{1}{2}$
	<hr/>			<hr/>
2 0	8 4 - $2\frac{1}{2}$		2 0	15 9 - $4\frac{1}{2}$
	<hr/>			<hr/>
	£4-4- $2\frac{1}{2}$ Ans			£7-19- $4\frac{1}{2}$ Ans.

* There is a sort of *mechanical* method of doing sums, casting up the total of the price of any commodity, which is often used in business by such as are not acquainted with taking the parts according to the strict order of this rule.

mechanical or customary Way of casting up any of the foregoing Examples for those that cannot divide by the Parts.

Let us take the last example, viz. 85 ells, at $22\frac{1}{2}d.$ per ell.

Rule. Count how many shillings, six-pences, groats, pence, or any other denomination the given number makes, and set the products under one another, as in addition; then add all together, and you have the answer.

Thus, the last example, 85 ells at $22\frac{1}{2}d.$ per ell.

85 shillings	} is {	£4 - 5 -
85 six-pences		2 - 2 - 6
85 three-pences		1 - 1 - 3
85 pence		- 7 - 1
85 half-pence		- 3 - $6\frac{1}{2}$

£7 - 19 - $4\frac{1}{2}$ as before.

Tyro. I heartily thank you, sir, for these observations.

Philo. I do it to serve you, take which way you please; for that is best which is soonest understood though it be a little deviating from the stated rule itself. And now, *Tyro*, we will proceed to the other part of this rule.

C A S E 4.

Rule 1. When the given price is above two shillings then (by *table 2*) take such shillings as are even parts of a £. and divide the given quantity thereby; then take the pence as are parts of a shilling, and the farthings as are parts of a penny, and divide each sum or line of the parts to which they belong. Or,

Rule 2. Multiply the given quantity by the shilling then take the parts for the pence and farthings, as in the last cases, and add all together; then divide by the shilling and it is done.

A few EXAMPLES, with Words at length.

		Yards
25	$\frac{1}{10}$	125 at 2s. 8½d. yd.
6d	$\frac{1}{4}$	12 - 10s.
2d	$\frac{1}{2}$	3 - 2 - 6d.
	$\frac{1}{2}$	1 - 0 - 10
		- 5 - 2½
		£16 - 18 - 6½ Ans.

Here 2s. is the $\frac{1}{5}$ of a £. therefore I divide 125 by 5 and it gives £.12-10s. the price at 2s. per yard. Then 6d. is the $\frac{1}{4}$ of 2s. I therefore divide £.12-10s. by 4, and it gives £.3-2-6. Then 2d. is the $\frac{1}{2}$ of 6d. I therefore divide £.3-2-6 by 3, and it gives £.1 and 10 pence, the price at 2d. per yard. Lastly, I say, 1 halfpenny is the $\frac{1}{4}$ of 2d. and take the $\frac{1}{4}$ of £.1-10d. which is 5s. 2½d. the price at a halfpenny a yard. The sum of all these is £.16-18-6½ Ans.

PROOF by the second RULE.

Yards	125 at 2s. 8½d.	2	250
			62 - 6d.
			20 - 10
			5 - 2½
210	3318 -	6½	
£16-18-6½ as before			

* Here I multiply 125 by 2, and it gives 250 shillings, at 2s. a yard. Then I say, 6d. is $\frac{1}{2}$ of a shilling, therefore I divide the top line 125 by 2, (and not 250) and it gives 62s. 6d. and so for any other sum; then I take the other parts, as before directed, as by the work appears.

Philo. Do you understand it?

Tyro I do very well; but I like the last way the best.

Philo. Take which you please; 'tis only use that makes both easy.

Ells	417 at 5s. 10d.	4	104 - 5s.
			17 - 7 - 6d.
£121-12-6 Ans.			

First, 5s. is $\frac{1}{4}$ of a £. I divide 417 by 4, gives £. 104 - 5s. then 10 pence is $\frac{1}{6}$ of 5s. therefore I divide £. 104 - 5s. by 6, and find the quotient £. 17, and 2 over; this 2 I call 2 pounds, and the odd 5s. makes £. 2 - 5, or 45 shillings; then I say, the 6's in 45 is 7 times, and 3 over; this I call 3 shillings, and divide that also by 6, it gives 6 pence.

P R O O F.

4d $\frac{1}{3}$ 417 ells at 5s. 10d.

5
 2085
 4d $\frac{1}{3}$ 139
 2d $\frac{1}{2}$ 139
 69 - 6d.
 2|0 243|2-6

£121-12-6 as before

Here I multiply by 5 and it makes shillings then I say, 4 pence is $\frac{1}{3}$ of a shilling, and take it out of the top line 417: I say again 4 pence is $\frac{1}{3}$, and set down the same; then 2 pence is $\frac{1}{6}$ of 4 pence, and having added all up, and divided by 20, have the same answer.

Two more EXAMPLES.

10s $\frac{1}{2}$ 57 pieces at 12s 9 $\frac{1}{2}$ d
 s.d.
 2-6 $\frac{1}{4}$ 28 - 10
 3d $\frac{1}{10}$ 7 - 2 - 6
 $\frac{1}{2}$ $\frac{1}{6}$ - 14 - 3
 - 2 - 4 $\frac{1}{2}$

£36-9-1 $\frac{1}{2}$ Ans.

P R O O F.

6d $\frac{1}{2}$ 57 pieces at 12s 9 $\frac{1}{2}$ d
 12
 684
 28 - 6
 14 - 3
 2 - 4 $\frac{1}{2}$
 2|0 72|9-1 $\frac{1}{2}$

£36-9-1 $\frac{1}{2}$ as before

10s $\frac{1}{2}$ 185 load at 18s 10d
 4s $\frac{1}{3}$ 92 - 10
 4s $\frac{1}{10}$ 37
 8d $\frac{1}{6}$ 37
 2d $\frac{1}{4}$ 6 - 3 - 4d.
 1 - 10 - 10

£174-4-2 Ans.

P R O O F.

6d $\frac{1}{2}$ 185 load at 18s 10d
 18 multiply

3330
 92 - 6
 61 - 8
 2|0 348|4-2

£174-4-2 as before

And thus, Tyro, you may take which method you please; but in the next case you are confined to take the parts according to table 2.

CASE 5.

When the given price of any thing is more than 20 shillings, or one pound, and not so much as 2 pounds, then let the given quantity stand without drawing a line under it, and that is the price at 1 pound; then take the parts for the shillings and pence, as in the last case, and the work is done.

EXAMPLES.

Ells	
45 at	£1-12-6
22 -	10
5 -	12-6
<hr/>	
£73 -	2-6 Ans.

Here I let 45 stand without drawing any line under it, which is the price at 1 pound *per* ell; then I take the parts according to the rule in the last case, and adding all together, have £.73 - 2 - 6 *Ans.*

Tons	
75 at	£1-17-11½
37 -	10
18 -	15
9 -	7-6
1 -	11-3
-	3-1½
<hr/>	
£142 -	6-10½ Ans.

Here you see 75 tons at £.1 *per* ton is £.75, which I let stand without drawing any line under it, and then I take the parts for 17s. 11½*d.* as you find in the margin.

CASE 6.

When the given price of the commodity is above 2 pounds, then multiplying by the pounds gives the pence, for the number of pounds; and the shillings, pence and farthings must be taken out of the top, or the number, as before directed.

EXAMPLE.

10s	$\frac{1}{2}$	37 loads at £3-16-9
		3
		<hr/>
		111
5s	$\frac{1}{2}$	18 - 10
1s	$\frac{1}{2}$	9 - 5
6d	$\frac{1}{2}$	1 - 17
3d	$\frac{1}{2}$	18 - 6
		9 - 3
		<hr/>
		£141 - 19 - 9 Ans.

Here I multiply 37 by £. 3, which gives £. 111 and then I say, 10s. is $\frac{1}{2}$ of a £. and take it off of the top £. 37, which is £. 18 - 10s. then I say 5 shillings is $\frac{1}{2}$ of 10, and take the $\frac{1}{2}$ of £. 18 - 10 which is £. 9 - 5s. and I go on taking the part as the work plainly shew

One EXAMPLE at large.

		Hhds.
10s	$\frac{1}{2}$	15 at £12-18-0 $\frac{3}{4}$
		12
		<hr/>
		180
5s	$\frac{1}{2}$	7 - 10
2-6	$\frac{1}{2}$	3 - 15
6d	$\frac{1}{2}$	1 - 17 - 6
$\frac{3}{4}$	$\frac{1}{2}$	7 - 6
		11 $\frac{1}{4}$
		<hr/>
		£193 - 10 - 11 $\frac{1}{4}$ Ans.

Here 15 hhds. at £. per hhd. is 180£; then I say, 10s. is $\frac{1}{2}$, and take $\frac{1}{2}$ of £. 15, the top number, which is £. 7 - 10s. and so proceed, by taking the rest of the parts, saying 5 shillings is the $\frac{1}{2}$ of 10 shillings, and take the £. 7 - 10s. &c.

And now, *Tyro*, I think you have had sufficient instructions in this rule, if you carefully observe the same. However, that I may omit nothing that may be useful to you in business, there are some shorter methods of casting up things than the rule itself teaches; but this is according to the given price, that is, when the given price is just two shillings, or when it is any even number of shillings, as in the two following cases.

CASE 7.

When the given price is any even number of shillings, as 2, 4, 8, 14, 16, &c. then multiply the given quantity by half the even number of shillings, and

off the first figure in the units place with a dash of your pen, or pencil: then all the figures to the left hand are pounds, and the figure you cut off in the units place being doubled, will be shillings, and you have the answer required.

EXAMPLES.

48 yards, at 4s. per yard.

Here the price is 4 shillings per yard, therefore I multiply the given quantity

£ 9 | 6 48 by half the price, (*viz.* 2) and it is

Ans. £ 9 - 12s 96; then I cut off the first figure, (*viz.* 6) and double it, which is 12 for the shil-

lings, and the figure 9 is the pounds; so is the answer.

£ 9 - 12s. Again,

328 ells, at 12s. per ell.

6

Here I multiply by 6, which is $\frac{1}{2}$

the given price, and cut off the first

£ 196 | 8

Ans. £ 196 - 16s.

figure, and double it for the shil-

lings, which is 16, and the figures

on the left hand of the dash are pounds.

More EXAMPLES.

417 yards at 14s.

Again, 695 ells at 18s.

7

£ 291 | 9 Ans. £ 291 - 18s.

9

£ 625 | 5 Ans. £ 625 - 10

CASE 8.

When the given price is any even number of shillings, and you would know what quantity of any thing may be bought for any even number of pounds sterling, then only add a cypher to the given pounds, and divide that sum by half the number of the given price, gives the answer in pounds sterling.

EXAMPLES.

How many *lb.* of tea, at 12*s.* *per lb.* may I have for £.36? *Ans.* 60 *lb.*

To £.36 add a cypher, and it makes 360

$\frac{1}{2}$ of 12*s.* is 6)60 *lb.* *Ans.*

How many gallons of rum, at 8*s.* *per gallon*, may I have for £.250?

To £.250 add a cypher, and it makes 2500

$\frac{1}{2}$ of 8*s.* is 4)625 *gall.* *Ans.*

Tyro. I like this very well, sir; and it is not only easier, but much shorter than taking the parts; but suppose there be fractions after the given number, how then?

Philo. Take such *fractional* parts out of the given price, and add it to the rest of the work, as by the following *case*.

CASE 9.

When the given quantity or number has *fractional* parts after it, then take such parts out of the given price, and place the sum under the rest, and add all together. Or thus, (which may be easier) multiply the price by the *numerator*, (or top figure of the fraction) and divide the product by the denominator, (or lower figure) and you have the *fractional* value at once.

10 <i>s.</i>	$\frac{1}{2}$	15 $\frac{5}{8}$ yards at 10 <i>s.</i>	Here I say, $\frac{1}{8}$ of
		<hr/>	10 <i>s.</i> is 1 <i>s.</i> 3 <i>d.</i> there
	$\frac{5}{8}$	7 - 10	fore $\frac{5}{8}$ is 5 times
		<hr/>	as much, <i>viz.</i> 6 <i>s.</i> 3 <i>d.</i>
		6 - 3 for $\frac{5}{8}$ ths	Or thus, I multiply
		<hr/>	10 <i>s.</i> the price, by
		£7 - 16 - 3 <i>Ans.</i>	5, the numerator
			which is 50, and dividing by 8, the denominator, I have
			6 <i>s.</i> 3 <i>d.</i> the price at $\frac{5}{8}$ ths.

38
9*
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rule is,

$3\frac{7}{8}$ yards at 18s.
9*

2 - 14

15 - 9

£ 3 - 9 - 9 *Ans.*

* Here the price being an even number of shillings, I multiply by $\frac{1}{2}$ of it, viz. 9, (according to *case 7*) and double the first figure, which is £2-14s. Then for $\frac{7}{8}$ I multiply the price, viz. 18s. by 7, and divide by 8, and it is 15s. 9d. And thus

may many sums in this *case*, as well as in others, be done in a very short and easy manner, by care and observation.

Tyro. I see it plainly, sir; and now, if you please, I could be glad to have a notion of working *weight*, viz. *qrs. cwt. lbs. &c.*

Philo. I am as ready to instruct you.

SECTION II.

Of WEIGHT.

ro. WHAT is the nature of working this rule?

Philo. By taking the parts, as before directed, as you will plainly see by and by; but first of all two following tables ought to be learned perfectly.

TABLE 1.

Even Parts of a Ton.

qrs.		
0	} is {	$\frac{1}{2}$ a ton.
0		$\frac{1}{4}$
0		$\frac{1}{5}$
2		$\frac{1}{8}$
0		$\frac{1}{10}$
0		$\frac{1}{16}$
1		$\frac{1}{20}$
0		$\frac{1}{25}$

TABLE 2.

Even Parts of a Cwt.

qrs. lb.		
2 - 0	} is {	$\frac{1}{2}$ an Cwt.
1 - 0		$\frac{1}{4}$
16		$\frac{1}{7}$
14		$\frac{1}{8}$
8		$\frac{1}{14}$
7		$\frac{1}{16}$

Having got the above tables pretty readily by heart, the rule is,

RULE.

R U L E.

Multiply the given tons, or hundreds, by the given price, and if there be any parts of shillings and pence work with them as in the foregoing examples; the being done, take the parts for the odd weight out of the given price, and place it under the rest, and the work is done.

But this is better understood by an example or two

What comes 7 C. 3 qrs. to, at £. 2 - 6s. per Cwt.

		2	
5s.	$\frac{1}{4}$	14	
1	$\frac{1}{5}$	1 - 15	
2 qrs.	$\frac{1}{2}$	- 7	
1	$\frac{1}{2}$	1 - 3	
		- 11 - 6	
		<hr/>	
		£ 17 - 16 - 6	<i>Ans.</i>

Here I multiply 7 by 2, and it gives £. 14; then I say, 5s. is $\frac{1}{4}$, and take $\frac{1}{4}$ of £. 7, the top number; then I say 1s. is $\frac{1}{5}$ of 5s. and take the $\frac{1}{5}$ of £. 1 - 15s. The being done, I begin saying, 2 qrs. is $\frac{1}{2}$

an Cwt. and I take the $\frac{1}{2}$ of the price £. 2 - 6s. and it is £. 1 - 3s. then 1 qr. is $\frac{1}{2}$ of 2 qrs. I take the $\frac{1}{2}$ of £. 1 - 3s. which is 11s. 6d.

What cost 14 ton, 6 C. 3 qrs. 14 lb. at £. 12 - 10s. per ton

		12	
10s.	$\frac{1}{2}$	168	—The value at £. 12 per ton.
5 C.	$\frac{1}{4}$	7	—Ditto at 10s. per ton.
1 C.	$\frac{1}{5}$	3 - 2 - 6	—The $\frac{1}{4}$ th of £. 12 - 10s.
2 qrs.	$\frac{1}{2}$	12 - 6	—The price of 1 C.
1 qr.	$\frac{1}{2}$	6 - 3	—Ditto of 2 qrs.
14 lb.	$\frac{1}{2}$	3 - 11 $\frac{1}{2}$	—Ditto of 1 qr.
		1 - 6 $\frac{3}{4}$	—Ditto of 14 lb.
		<hr/>	
		£ 179 - 5 - 11 $\frac{1}{4}$	<i>Ans.</i>

I have here, *Tyra*, set against every line what value is, and where it is taken out of, that you may the better grounded in the work: for remember, money is taken out of the given weight, and the

weight is taken out of the given price, as you were told in the rule.

Tyro. I see the nature of it, sir, plainly.

Philo. Then I shall leave you a few questions for you to work, or prove at leisure.

QUESTIONS to exercise the Learner.

1. What cost 125 lb. of indigo, at $14\frac{3}{4}$ d. per lb?

Ans. £.7 - 13 - $7\frac{3}{4}$.

2. What is gained in £.476, at 4s. $10\frac{1}{2}$ d. per £?

Ans. £.116 and 6d.

3. What is the value of 47 load, at £.3 - 15 - $6\frac{1}{2}$ per load? *Ans.* £.177 - 10 - $5\frac{1}{2}$.

4. What cost 9 ton, 15 C. 3 qrs. 27 lb. of iron, at 14 per ton? *Ans.* £.137 - 3 - $10\frac{1}{2}$.

D I A L O G U E X.

SECTION I.

SIMPLE INTEREST.

SIR, you have given me great satisfaction in the rules of *Practice*, and now I should be glad to be informed something concerning *Interest*. May I know what it is?

Philo. *Interest* is money that arises from a certain sum lent out at so much *per cent. per annum*; and is a consideration allowed by the borrower to the lender.

Tyro. What do you mean by *per cent. per annum*?

Philo. *Per cent.* signifies by 100 pounds, and *per annum* signifies by the year, or 12 calendar months.

Tyro. How is the interest found?

Philo. *Interest* may be performed by the *Rule of Three direct* (as you may see by *example* 1 following) but in some cases it is done easier and shorter by *Practice* and custom,

custom, and that is the reason that *Interest* has a place and title in all books of arithmetic by itself.

Tyro. Please to give an example by the *Rule of Three* and prove it by the short customary way.

Philo. I will.

EXAMPLE.

What is the interest of £.175 - 10s. for a year, £.5 per cent. per annum?

£.	£.	£.	s.
If 100—be	5 —	what is	175 - 10
20		20	
<hr/>		<hr/>	
2000		3510	
		5	
		<hr/>	
		2000)17550(8£.	
		16000	
		<hr/>	
		1550	
		20	
		2000)31000(15s.	
		2000	
		<hr/>	
		11000	
		10000	
		<hr/>	
		1000	
		12	
		<hr/>	
		2000)12000(6d.	
		12000	
		<hr/>	
		0	

Ans. £.8 - 15 - 6.

Here you see I am obliged to make my *first* and *second* numbers of one name, so that according as

is, it requires a great many figures; but by the
er method you spare all this trouble; which pray
erve.

Second METHOD. CASE 1.

year, Multiply the principal, or given sum, by the rate
cent. and cut off two figures towards the right-hand,
which is the same, observe, as dividing by 100) and
figures towards the left are pounds interest: this
e, multiply the figures cut off to the right-hand by
take in the odd shillings, and cut off two figures,
efore, and the figures on the left-hand are shillings:
n multiply the remainder by 12, and cut off two
res, and the figures on the left-hand are pence.
ly, multiply by 4, and cutting off as before, you
e the farthings.

Thus the above example is done, as under.

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \\
 175 - 10 \\
 \quad \quad 5 \\
 \hline
 \text{£.} 8) 77 - 10 \\
 \quad 20 \\
 \hline
 \text{s.} 15) 50 \\
 \quad 12 \\
 \hline
 \text{d.} 6) 00
 \end{array}$$

Ans. £. 8 - 15 - 6 as above.

ra. Indeed this is very short to what the first
x is.

oilo. Then pray observe the rule; for all the fol-
ing examples are performed after the same manner;
remember, that cutting off two figures towards the
t-hand is the same as dividing by 100.

Quest.

Quest. 2. What is the interest of £.515 - 10s. for 1 year, at £.4 per cent. per annum?

£.515 - 10

4 per cent. multiply

£.20)62

20

s.12)40

12

d.4)80

4

qrs.3)20

Ans. £.20 - 12 - 4 $\frac{1}{4}$ $\frac{20}{100}$, or $\frac{2}{10}$ ths of a farthing.

Quest. 3. What is the interest of £.1050 for 1 year, at £.3 per cent. per annum?

£.1050

3 per cent.

£.31)50

20

s.10)00 *Ans.* £.31 - 10s.

Tyro. You need not give me any more example this sort; but suppose the money be lent for more one year, how then?

C A S E 2.

Philo. When the given rate per cent. is an even of £.100, then divide the principal by that even and the quotient is the answer at once.

Take example 1, viz. £.175 - 10s. at £.5 per cent.

Here £.5 being $\frac{1}{20}$ of £.100, I only divide £.175 by 20, and it is done.

£. s.
20)175 - 10*

£.8 - 15 - 6 *Ans.*

* *Note,* After I divide 175 by 20, I say, the part of 10s. is 6d.

Quest. 4. What is the interest of £.345 - 17 - 6, at 25 per cent?

Here £.25 being $\frac{1}{4}$ of £.100, I divide the principal 345 - 17 - 6 by £.4, and it is done.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 4 \overline{) 345 - 17 - 6} \\ \hline \text{£.} 86 - 9 - 4\frac{1}{2} \text{ Ans.} \end{array}$$

C A S E 3.

When the principal is put out for years and months, find the interest for the years first, and take the months even parts of a year, according to the rules of *Prac-*

Quest. 5. What is the interest of £.175 - 10s. for 3 years, 9 months, at £.5 per cent. per annum?

$$\begin{array}{r} \text{£.} 175 - 10 \\ \hline \text{5 per cent.} \end{array}$$

$$\begin{array}{r} \text{£.} 8 \overline{) 77 - 10} \\ \hline 20 \end{array}$$

$$\begin{array}{r} \text{s.} 15 \overline{) 50} \\ \hline 12 \end{array}$$

$$\begin{array}{r} \text{d.} 6 \overline{) 00} \end{array}$$

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{Interest for 1 year is } 8 - 15 - 6 \\ \hline 3 \end{array}$$

$$\begin{array}{l} 6 \text{ mths. } \frac{1}{2} \text{ a year} \\ 3 \text{ mths. } \frac{1}{4} \end{array} \left| \begin{array}{r} 26 - 6 - 6 \text{ for 3 years} \\ 4 - 7 - 9 \\ 2 - 3 - 10\frac{1}{2} \\ \hline \text{Ans. } 32 - 18 - 1\frac{1}{2} \end{array} \right.$$

A practical

A practical QUESTION.

Quest. 6. A gentleman left his niece by will £.558 - 15s. to be paid her when she came to age, with interest at £.4 per cent. Now she came to age in years, 9 months, and 3 weeks: what has she got to receive in all, principal and interest?

$$\begin{array}{r}
 \text{£.558} - 15\text{s.} \\
 \hline
 \text{4 per cent.} \\
 \text{£.22)35} \\
 \quad 20 \\
 \hline
 \text{s. 7)00}
 \end{array}$$

Interest for 1 year £.22 - 7s.
5 years

	$ \begin{array}{r} 111 - 15 \\ 11 - 3 - 6d. \\ 5 - 11 - 9 \\ 18 - 7\frac{1}{2} \\ 9 - 3\frac{1}{4} \end{array} $	
6 mths. $\frac{1}{2}$ year		
3 months, $\frac{1}{4}$		
2 weeks $\frac{1}{6}$		
1 week $\frac{1}{2}$		
	$ \begin{array}{r} \text{£.129} - 18 - 2\frac{1}{4} \text{ interest} \\ 558 - 15 \text{ legacy} \end{array} $	

The legacy and interest is £.688 - 13 - 2 $\frac{1}{4}$

Tyro. Sir, I thank you for these two useful examples, and now I would only ask this one favour, and that is how am I to tell the interest of any sum for any time, 3 $\frac{1}{2}$, 4 $\frac{1}{2}$, 4 $\frac{3}{4}$, or the like per cent?

Philo. It is something more difficult than the other, but an example or two, with your due observation, will make it very easy.

C A S E 4.

When the given rate *per cent.* is not even pounds, it is $2\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{2}$, or the like, then multiply the principal or given sum, by the even pounds, and take the $\frac{1}{4}$, or $\frac{3}{4}$ of the said principal, or given sum, and add to that product; then cut off two figures, and proceed in all respects as before.

Quest. 7. What comes the interest of a bond of £85 to, at £.4 $\frac{1}{2}$ (that is, £.4 - 10s.) *per cent. per annum?*

$$\begin{array}{r}
 \text{£.} 685 \\
 \quad 4 \\
 \hline
 \begin{array}{l}
 \text{s. } 10 \quad \left| \quad \frac{1}{2} \right| \begin{array}{l} 2740 \\ 342 - 10 \\ \hline \text{£.} 3082 - 10 \\ 20 \\ \hline \text{s. } 1650 \\ 12 \\ \hline \text{d. } 600 \end{array}
 \end{array}
 \end{array}$$

Here I multiply by 4; then for $\frac{1}{2}$ *per cent.* which is 2 shillings, I say, 10s. is $\frac{1}{2}$, as in *Practice*, and take of 685, and add to the other, it is £.3082 - 10s. then cut off two figures, thus, 30)82, and proceed as before, I find it £.30 - 16 - 6.

Quest.

Quest. 8. Lent £.500 upon a mortgage, to receive interest at $4\frac{1}{2}$ (*viz* £.4 - 15s.) *per cent.* till it was paid off: now the mortgage was paid off 3 years 8 months after: I demand the interest due to me?

$$\begin{array}{r}
 \text{£.} \\
 500 \\
 4 \\
 \hline
 \begin{array}{l}
 \text{s.} \left| \begin{array}{l} 10 \\ 5 \end{array} \right| \begin{array}{l} \frac{1}{2} \\ \frac{1}{2} \end{array} \left| \begin{array}{l} 2000 \\ 250 \\ 125 \end{array} \right. \\
 \hline
 \text{£.23)75} \\
 20 \\
 \hline
 \text{s. 15)00}
 \end{array}$$

Interest for 1 year is £.23 - 15
multiply by 3

$$\begin{array}{r}
 \begin{array}{l} 6 \text{ months} \\ 2 \text{ months} \end{array} \left| \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \end{array} \right| \begin{array}{l} 71 - 5 - \\ 11 - 17 - 6 \\ 3 - 19 - 2 \end{array} \begin{array}{l} \text{for 3 yrs} \\ 6 \text{ mths} \\ 2 \text{ mths} \end{array} \\
 \hline
 \text{£.87 - 1 - 8} \text{ Ans.}
 \end{array}$$

Tyro. I understand all these examples very well; but suppose the interest be for less than a year, how then am I to find it?

Philo. Very easily.

C A S E 5.

When the interest of any sum is to be found for less than a year, *viz* for 1, 2, 3, 4, or 10 months, or for months and odd days, then first find the interest for 1 year, and take the parts of that year's interest according to the given time: thus, if it be 6 months interest, take the half of the year's interest; if 4 months, take $\frac{2}{3}$; if three months, take $\frac{1}{4}$; and for 2 months or odd weeks or days, take the parts of those months.

Qu.

Ques. 9. What is the interest of £.175 - 10s. for 8 mths, at £.5 per cent. per annum?

$$\begin{array}{r}
 \text{£. } 175 - 10 \\
 \hline
 \phantom{\text{£. }} 5 \\
 \hline
 \text{£. } 8) 77 - 10 \\
 \phantom{\text{£. }} 20 \\
 \hline
 \text{s. } 15) 50 \\
 \phantom{\text{s. }} 12 \\
 \hline
 \text{d. } 6) 00
 \end{array}$$

Interest for 1 year is $\begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 8 & - 15 & - 6 \end{array}$

Then say 6 mths. $\left| \begin{array}{c} \frac{1}{2} \\ \frac{1}{3} \end{array} \right| \begin{array}{ccc} 4 & - & 7 & - & 9 \\ 1 & - & 9 & - & 3 \end{array}$ for 6 months.
 2 mths. $\left| \begin{array}{c} \frac{1}{2} \\ \frac{1}{3} \end{array} \right| \begin{array}{ccc} 4 & - & 7 & - & 9 \\ 1 & - & 9 & - & 3 \end{array}$ for 2 months.
 $\text{£. } 5 - 17$ *Ans.*

For 3 yrs. 6 mths. 2 mths. are you see I take the half of one year's interest, which is £.4 - 7 - 9 for 6 months; then I say, 2 months of 6 months, and take the $\frac{1}{3}$ of £.4 - 7 - 9, and add together, is £.5 - 17 for 8 months.

Ans. I am satisfied, sir.

Ques. 10. This is the best way for calculating interest general, and will be near enough for common practice should there be weeks or days; for it is only the parts for such weeks and days, as was said

Ques. 11. But suppose the interest be put out for such number of days as cannot well be taken in their even how then?

R U L E 1.

Ques. 12. First find the interest for one year as before; then, if 365 days give so much, what will so many days give? Thus, suppose the interest for one year be

£.17

£.17 - 10 - 10, and it was required to find the interest for 73 days, I say,

If 365 days give £.17 - 10 - 10, what will 73 days give? *Ans.* 3 - 10 - 2.

R U L E 2.

Bring the principal, or money lent out, into pence and multiply those pence by the number of days the sum is put out at, or continues out; then if the rate *per cent.* it be put out at be £ 5, divide by 7300; if it be £.6 *per cent.* divide by 6083, you have the answer in pence. There are many other ways to find interest for days; but I would recommend the first of these two to the learner, as it serves for all rates *per cent.* and is most certain.

S E C T I O N II.

Of ASSURANCE, BROKERAGE, or COMMISSION

Tyro. **W**HAT is Assurance, Brokerage, or Commission? *Philo.* Assurances, or Insurances, are of various sorts. An Assurance is when any person agrees with another at a certain rate *per cent.* to insure his life for such a time, or his ship upon a voyage against the dangers of the seas, or his house or goods against fire. Brokerage is an allowance of so much *per cent.* given to Brokers, or persons employed in buying and selling stocks, or transacting business between buyer and seller: and Commission is also an allowance of so much *per cent.* for buying and selling any sort of commodity, by the order of any other person, &c.

Tyro. Then I perceive these are all cast up the same way as interest of money is.

Philo. They are so, only interest is so much *per cent. per annum*; but brokerage is cast up only

much *per cent.* ready money, without any regard to
me. Of these in their order.

1. Of ASSURANCES.

Quest. 1. What comes an assurance of £.580 to, at
10½ *per cent*?

$$\begin{array}{r}
 \text{£. } 580 \\
 \text{10} \\
 \hline
 \text{s. } 10 \left| \begin{array}{l} 5800 \\ 290 \\ 145 \end{array} \right. \\
 \hline
 \text{£. } 62) 35 \\
 \text{20} \\
 \hline
 \text{s. } 7) 00
 \end{array}$$

Ans. £.62 - 7

Here I multiply by 10, then I take the parts out of
top, or £.580, and add all together, and cut off two
ures (which is dividing by £.100) and proceed as in
erest, by multiplying the remainder by 20, &c.

Quest. 2. Shipped for *Jamaica* goods to the value of
200, upon which I made an assurance, at £.7½ths
cent. what does it come to?

$$\begin{array}{r}
 \text{£. } 1200 \\
 7 \\
 \hline
 \frac{4}{8} \text{ is } \frac{3}{2} \left| \begin{array}{l} 8400 \\ 600 \\ 150 \end{array} \right. \\
 \hline
 \text{£. } 91) 50 \\
 20 \\
 \hline
 \text{s. } 10) 00
 \end{array}$$

Here I multiply by 7. Then I say for the fraction,
ghths is ½, that is, ½ of 8ths, (for 8 eighths is equal
L to

218 ASSURANCE, BROKERAGE, COMMISSION, &c.

to 1 whole integer) and take the $\frac{1}{2}$ of the top number then I say, $\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{4}{8}$ ths; then I add all together, and cut off two figures, as before, &c. and find the answer $\pounds.91 - 10s.$

2. Of BROKERAGE and COMMISSION.

Tyro. You need not give me any more examples for I see the work is the same, though under different titles or names.

Philo. It is so; but still when the *per cent.* is shillings, it may be a little difficult to you, therefore will give you an example or two.

Quest. 3. What is the commission of a broker for buying or selling $\pounds.520$ stock, at half a crown *per cent.*

Rule. Multiply the money by the shillings, and take the parts for the pence, as in *Practice*; then add all together, and cut off two figures to the right-hand, and those towards the left are your answer in shillings.

$$\begin{array}{r}
 \pounds.520 \\
 \quad 2 \\
 \hline
 6d. \mid \frac{1}{2} \mid \begin{array}{r} 1040 \\ 260 \\ \hline \end{array} \\
 \hline
 s.13)00 \\
 \text{Ans. } 13 \text{ shillings.}
 \end{array}$$

Quest. 4. What is my commission on $\pounds.1500$ at *per cent*?

$$\begin{array}{r}
 7 \\
 \hline
 s.105)00 \\
 \text{Ans. } \pounds.5 - 5s.
 \end{array}$$

Another way.

Cut off two figures of the given sum, and multiply remainder by 20, 12, and 4, and this gives the answer

at £.1 per cent. Then take the parts of the sum of £.1 per cent. as in *Practice*, you have your answer.

Quest. 5. What is my commission on £.1252 - 10s. at 7s. 6d. per cent.

£.12)52 - 10s. At £.1 per cent. it is £.12 - 10 - 6

$$\begin{array}{r} 20 \\ \hline 20 \\ \hline \end{array}$$

 Then, 5s. $\left| \frac{3}{4} \right| 3 - 2 - 7\frac{1}{2}$

$$\begin{array}{r} 2 - 6 \\ \hline 2 - 6 \\ \hline \end{array}$$

$$\left| \frac{1}{2} \right| 1 - 11 - 3\frac{1}{2}$$

$$\hline$$

Ans. £.4 - 13 - 11 $\frac{1}{2}$

$$\begin{array}{r} 10 \\ \hline 10 \\ \hline \end{array}$$

 d.6)00

3. COMPOUND INTEREST.

Tyro What is *Compound Interest*?

Philo. *Compound Interest* is called *Interest upon Interest*; that is, the interest of the principal, and the interest of the interest, added together, is the *Compound Interest*. But it being seldom used without it be in purchasing of annuities, &c. and being very tedious to calculate, is done with more ease by *Decimal Fractions*.

Tyro. But I should be glad if you would give me a little better notion of it.

Philo. Suppose I put out £.100 for 3 years, at £.5 per cent. compound interest, what does it come to?

First, I find the simple interest for one year is £.5; then the second year I am to count what the interest £.105 is, and find it £.5 - 5. This I add to the last principal £.105, and it makes £.110 - 5. Then I see that is the interest of £.110 - 5s. and find it £.5 - 10 - 3 for the third year. Then I add these different interests together, viz. £.5, £.5 - 5s. and £.5 - 10 - 3, and it makes £.15 - 15 - 3, the *compound interest* for three years on £.100, at £.5 per cent. And thus for any number years, at any rate per cent.

SECTION III.

OF REBATE OR DISCOUNT.

Tyro. WHAT do you mean by *Rebate* or *Discount*?

Philo. *Discount* is when a sum of money, which is to be paid at any time to come, is satisfied by paying of it down in present money, which present money being put out after the same rate *per cent.* and for the same time, will be increased to the debt or sum that was first to be paid.

Tyro. Explain this a little fuller in other words.

Philo. Suppose a person owes me 100 guineas, *viz.* £.105, to be paid in a year, or 12 months to come; how much present money will satisfy the debt? *Ans.* £.100 present money will satisfy a debt of £.105 due 12 months hence; because £.100 put out to interest 12 months will be £.5.

There are several methods of performing this rule of *Discount*; but as it is best done by *Decimal Fractions*, I shall only give this one general rule.

A general RULE.

1. Add the interest of £.100 for the given time to £.100, and make this your *first number*; then place £.100 only for your *second number*, and the given debt for your *third number*, and the answer will be the present money that will satisfy the debt due twelve months to come; then take this present money from the debt, and the remainder is the *rebate* for twelve months.

2. Having thus found the *rebate* for one year, if the time be more or less than 12 months, take the parts of the rule of *Practice* accordingly.

3. Thus it appears that the *first* number is always known according to the rate *per cent.* for one year, as by the following table :

For at £.3 *per cent.* the *first* number will be 103, &c.

£. 3 <i>per cent.</i>	—	103
4	—	104
5	—	105
6	—	106
7	—	107
8	—	108

These, observe, are *first* numbers, according to the *rebate per cent.* for one year; £.100 is your *second*, and the debt your *third*, and the answer is the present money to be paid down.

Quest. 1. What present money will pay a draught or note of hand of £.368 - 4s. due three months hence, allowing rebate at £. 5 *per cent.* *per annum*?

If 105—100—368 - 4. *Ans.* 350 - 13 - 4, which taken out of the debt £.368 - 4s. leaves £.17 - 10 - 8, the discount for one year. See the work.

Rule. Bring the *first* and *third* number into shillings; then multiply the *third* by your *second*, and divide by your *first*, and you have £.350, and 14 remains, which also multiply by 20, and divide by your *first* number, you have 13 shillings, and 7 remains, which multiply by 12, and dividing by the same *first* number, you have 4d.

Note of hand £.368 - 4

Present money 350 - 13 - 4

Discount is | 17 - 10 - 8 for one year

3 months | $\frac{1}{4}$ | 4 - 7 - 8 *Ans.* for 3 months.

L 3

Which

Which taken out of £.368 - 4s. remains £.363 - 16 - 4 present money to be paid down.

Here you see for one year the *discount* is £.17 - 10 - 8, and as 3 months is $\frac{3}{4}$ of a year, I take the $\frac{3}{4}$ th of that sum, which is £.4 - 7 - 8. *Ans.*

Note. Had the note been drawn for 15 months, that is, 1 year and 3 months; then add £.4 - 7 - 8, the *discount* of three months, to £.17 - 10 - 8, which is £.21 - 18 - 4, the *discount* of £.368 - 4s. for fifteen months.

An EXAMPLE for Practice.

What present money will pay a debt of £.500, due 3 years, 4 months hence, allowing *rebate* at £.6 per cent. per annum? *Ans.* £.416 - 13 - 4. *Rebate* £.83 - 6 - 8. For by the table, your *first* number will be £.106 for 1 year, your *second* £.100, and the *third* the debt. And, for proof, find the interest of £.416 - 13 - 4 for three years and four months, at £.6 per cent. and add to it £.416 - 13 - 4, you will have £.500, the debt itself.

And, by these examples, you may find the *discount* at any rate per cent. and for any time, by observing the foregoing directions.

Tyro. I see the nature of working this rule very plainly.

Philo. Then I will give you a notion of another useful rule, *viz.*

SECTION IV.

EQUATION OF PAYMENTS.

Tyro. **W**HAT do you mean by *Equation of Payments*?

Philo. *Equation of Payments* is when a certain sum of money becomes due at different times, or is to be paid at different payments, and it is agreed between the debtor and creditor to set or pitch upon a certain time for the payment of the whole, without any damage or loss to either.

Tyro. This is very pretty, as well as useful: pray give me the rule for working questions of this sort.

R U L E.

Multiply the sum of each particular payment by the time it is to be paid in: then add the several products together, and divide the sum by the whole debt, and the quotient is the *equated*, or just time for the payment of the whole.

Quest. 1. A merchant buys goods to the value of £.140, £.30 of which is to be paid in 1 month, £.40 in 2 months, £.20 in 4 months, and £.50 in 6 months; but it is agreed upon an *equated* time to pay the whole at once; I demand the *equated*, or just time of payment?

I set down each payment, and the time right against it, and multiply the money by the time, and set the product of each right against them, as follows:

L 4

£.30

EQUATION OF PAYMENTS.

£.30 multiplied by 1 month is £.30
 40 — by 3 months is 120^{ps}
 20 — by 4 months is 80^{ps}
 50 — by 6 months is 300

Debt 140)530(3 months

420

110

30*

140)3300(23 days

280

500

420

80

Ans. 3 months, 23 days.

* *Note*, I multiply by 30, the days in a calendar month, according to the custom of business, because 12 such months are reckoned to the year.

Quest. 2. A merchant bought goods to the value of £.500, upon the following conditions, *viz.* $\frac{1}{4}$ th to be paid in 2 months, and the remainder in 6 months: I demand the equated time to pay the whole at one payment?

First, $\frac{1}{4}$ of £.500 is £.125 multiplied by 2 mths £.250
 The remainder is 375 multiplied by 6 mths 2250

Debt

£.500

5|00)25|00

Ans. 5 mths

Now £.2500 divided by the debt £.500 is 5 months for the answer, and thus for any other question. Do you understand it?

Tyro. There can be nothing easier to apprehend than this.

QUESTIONS

QUESTIONS to exercise this RULE.

3. A father left his son £.600, to be paid by his executor, as follows: $\frac{1}{3}$ in 2 months, $\frac{1}{3}$ in 3 months, and $\frac{1}{3}$ in 8 months; but he being very troublesome to the executor, and he as willing to be rid of him, they agreed upon a time for the payment of the whole at once. The equated time is demanded. *Ans.* 4th 3 months, viz. 4 months and 10 days.

4. A owes B £.2000, to be paid three months hence; but A agrees to pay him £.1200 down, provided he will give him a longer time for the payment of the remainder. The question is, how long time B ought to give him to pay the other £.800 in? *Ans.* 7 $\frac{1}{2}$ months.

Tyro. What other rules are there in common Arithmetic?

Philo. Several others, as *Allegation*, *Medial* and *Alternate*, *Single* and *Double Position*, *Composition of Medicines*, *Arithmetical* and *Geometrical Progression*, &c. but these being not so much required in business (or at least may be done by the *Rule of Three Direct*) I shall give a little notion of the two last, because it may be of service to you, in case your fancy should turn to any part of the mathematical studies.

DIALOGUE IX.

SECTION I.

Of ARITHMETICAL PROGRESSION.

Tyro. WHAT do you mean by *Arithmetical Progression*?

Philo. *Arithmetical Progression* is by some called *Comparative Arithmetic*, because in a series of numbers

one is compared with another, in order to know what ratio or proportion one bears to the other; that is, in what manner they differ, and how much, and that difference is always equal in numbers that are continued.

OBSERVATION.

1. Let us take any numbers that differ alike from one another: thus, 3, 6, 9, 12, 15, &c. differ by 3, called the common excess, difference, or ratio. Also, 1, 9, 17, 25, 33, are numbers in *Arithmetical Progression*, whose ratio or common difference is 8.

2. In any series of figures in *Arithmetical Progression*, when the number of places is odd, viz. 3, 7, 11, 15, 19, or the like, the double of the middle figure, or place, is equal always to the sums of the first and last figure; thus, 2, 6, (10) 14, 18. The double of the middle term 10 is equal to the sum of the first and last number, viz. 20. So 3, 8, 13, (18) 23, 28, 33. The middle term 18 doubled is equal to the sum of 3, and 33, viz. 36.

Tyro. How many cases are there in this rule?

Philos. Six; but two of them will be sufficient for your purpose, in working questions in general, or to give you a true notion of the nature of the rule itself.

CASE I.

The number of places or terms, and the ratio, or common excess, being given, to find that last number:

Multiply the number of places less by 1, by the ratio, or common excess, and to that product add the first number, and that sum will be equal to the last number.

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1. Let the first number be 3, number of places 7, and common excess 5, what is the last number?

Here the number of places is 7, therefore I count them for 6 (which is less 1) and the *ratio*, or common excess is 5, now five times 6 is 30; then I add the first number, *viz.* 3, and it is 33, the last number, and so on, if the places were ever so many.

2. Let the number of terms be 121, the first number 5, and the common excess or *ratio* 9; I demand the last number?

Here the number of places less by 1 is 120, which multiplied by 9, the *ratio*, gives 1080, to which add the first number 5, gives 1085, the last number. And this you may prove by setting them down thus, 5, 14, 23, 32, 41, till you have 121 places, and your last number will be 1085. But how tedious is this! since you see the last number is so easily found.

A QUESTION for Exercise.

A traveller set out, and the first day went 6 miles, the second day 9, increasing every day's journey 3 miles, and travelled 61 days; how many miles did he go the last day? *Ans.* 186 miles. See the next Case.

C A S E 2.

The extremes, that is, the first and last number, and the number of terms being given, to find the aggregate, or total sum of all the series of numbers:

Add the first and last numbers together, and multiply that sum by half the number of places, and the product will be the sum of all the series. Or, in case the number of terms be odd, then add the first and last numbers together, and multiply that sum by the number of the terms, and divide that product by 2, and you have the total of all the places.

Thus, in the last question concerning the travels, the first number is 6, the last number 186 miles, and

the places 61. Therefore, by the rule, add the first number 6, and the last number 186 together, and it is 192, which I multiply by 61, and it is 11712, and dividing by 2, I have 5856, the miles he went in all, in 61 days.

A QUESTION for Exercise.

Quest. 2. Suppose 100 stones be placed in a straight line, at a yard distance from each other, and a person undertakes to pick up one at a time, and bring it back to the place where he first set out; how far will he have gone when he has picked up the whole?

Here the number of places are 100, and as the stones are but one yard from each other, the common excess is but 1, and therefore the last number is also 100. But here you are to consider, the man is to come back with every stone; therefore, when he fetches the first stone, in coming back he goes two yards to put in the first stone, therefore 2 is your first number, and by the same rule 200 will be your last number.

Last number	200
First number	2
	<hr style="width: 50px; margin: 0;"/>

202

Half of the number of places	50
	<hr style="width: 50px; margin: 0;"/>

He goes in all — 10100 Yards

Which reduced into miles, is $5\frac{1}{2}$ miles, and 420 yards which wants but 20 yards of $5\frac{3}{4}$ miles.

Quest. 3. Suppose stones were laid one yard asunder in a right line, for one mile in length, and to be picked up one by one, coming back with one at a time; how far would the person go that performs it?
Ans. 1761 miles.

SECTION II.

Of GEOMETRICAL PROGRESSION.

I Am mightily pleased with what you have shewn me; but pray what is *Geometrical Progression*, wherein does it differ from the other?

Philo. *Geometrical Progression*, or *Proportion*, is when numbers differ from one another, by the like *ratio*, or *son*, as in *Arithmetical Progression*, only with this difference, that the former is the effect of *Addition*, and the latter of *Multiplication*, all the numbers having one common multiplier. As 2, 4, 8, 16, differ by double *son*, or the *ratio* is 2; for twice 2 is 4, twice 4 is 8, &c. So also, 1, 7, 49, 343, &c. Here the *ratio* is 7, for 7 times 1 is 7, and 7 times 7 is 49, &c. So that we see it is *Multiplication*.

Tyro. I thank you, Sir: What else, pray?

NOTE 1.

Philo. Any three numbers differ in *Geometrical Progression*; the product of the *extremes*, that is, the product of the *first* and *last* numbers, is equal to the square of the *mean*, that is, equal to the middle number multiplied by itself.

Let 4, 12, and 36 be the 3 numbers, whose *ratio* is 3, that is, each is 3 times more than the foregoing number; then will the product of the extremes 4 and 36 be equal to the square of 12, the *mean*, viz. equal to 144. So also 2, 16, and 128. Here 128 multiplied by 2 is equal to 16 multiplied by 16 equal to 256.

NOTE 2.

Any four numbers that differ in *Geometrical Proportion*, either continued or interrupted (provided the interruption or breaking off be between the *second* and *third* numbers)

numbers) the product of the *means* is equal to the product of the *extremes*; that is, the product of the *first* and *fourth* is equal to the product of the *second* and *third* numbers.

Let the 4 numbers be 3, 12, 48, and 192, whose *ratio* is 4. Here 192 multiplied by 3, the *first* number is equal to 48 by 12, viz. equal to 576.

Again, Suppose the numbers be interrupted, as 5, 448, and 2688. Here the *second* term is 6 times more than the *first* term, and the *fourth* term 6 times more than the *third*; therefore, 2688 multiplied by 5 is equal to 448 multiplied by 30.

Note, This mark (\div) stands for *Geometrical Progression*.

NOTE 3.

The *ratio* of any numbers in \div is found by dividing any consequent by its antecedent, that is, dividing the last number by the foregoing number. Thus the *ratio* of the last numbers is found by dividing 30 by 5, or 2 by 448, which gives 6, the *ratio*.

Tyro. I understand this very well; but in a long series of numbers, how am I to find the last number?

Philo. This is very easily done, and as you have the rule in many, or, at least, have it but for certainty, *ratio*, or difference of numbers, I shall give you a standing rule; be the first number and the *ratio* what you will.

CASE I.

To find the last number in any series in \div having the *first* number and *ratio* given, set down your first number, and multiply it by the *ratio*, and that product again by the *ratio*, or common difference, and thus on for 5 or 6 terms, at pleasure; then multiply all those places by itself, and divide the product thereof by the *first* number, and it will be the double of the last number, wanting one place. Thus, suppose you

by the 6th place by itself, and divide by the *first* number, the quotient will be the 11th place; then if you multiply the 11th place by itself, and divide by the *first* number, the quotient will be the 21st place, which is double of 11, wanting 1.

N. B. When the first term is unity, or 1, there is no occasion to divide; for having multiplied by the same a few times, as before directed, (suppose as far as the 6th place) this term multiplied by itself will be the product of the double of terms wanting 1, as was said before.

ro. I am obliged to you, Sir; but how shall I find the sum of all these series?

hilo. By the next case.

C A S E 2.

When the ratio (or common difference) and the last number are given, to find the aggregate or total sum of all the terms.

Multiply the last term, or number, by the common difference, or *ratio*, and from that product take the *first* number: then divide the remainder by the common difference, less 1, and the quotient will be the sum of all the series. Or,

Multiply the *second* and last term together; then multiply the *first* term into itself, and take that product from the product of the *second* and last term; then divide the remainder by the *second* term, less 1, the *first* term will give the sum of all the series.

ro. Please to give me one example.

hilo. I will give you a practical question to exercise these cases.

ro. A cunning jockey had a fine gelding, to which a Gentleman took a particular fancy, and after some words had passed between them, the jockey offered to sell him to the gentleman for the price which his horse would come to, at one farthing for the *first* nail, and double the price every nail. Now the number of nails in the gelding's shoes were 28, I demand then how much he was sold for at this rate?

Now,

Now, observe, *Tyro*, here the common excess is (that is a farthing a nail doubled :) Now to find the last number by *Case 1*, double a few places, suppose as far as the 7th, which will be 64; then multiply by itself, viz. 64, you have 4096, the 13th place, which doubled gives the 14th place, viz. 8192. This multiplied by itself gives the 27th place, which doubled gives the 28th or last place, viz. 134217728 farthings; then proceed according to *Case 2*, you will find the sum of all the series to be 268435455 farthings, which is £.279620-5-3 $\frac{3}{4}$, a sum too large for the ignorant purchaser. And had there been but four nails more in shoes, (viz. 32) he would have come to £.4473924-

* It is no easy task to make some persons believe the truth of the increase of figures in Geometrical Progression in these questions; but suppose a servant to agree with his master to serve him thirty years, 1 single wheat corn the first year, for the second 10, for the third 20, and so on. The produce of wheat would be more than all the ships in England could carry away at once; and the money for his wages would be more than all the land could pay, if at 30 years purchase for ever.

SECTION III.

Of PERMUTATION, or Variety of Changes

Tyro. I Imagine this is much after the same manner as *Progression*; is it not?

Philo. This shews the increase of numbers as the other; but does not teach you to find the sum of all, only their different variations or changes follows:

Multiply every number together, that is, the first by the second, and that product by the third figure, and so on till you have gone through all the given numbers; the last product is the variety of changes.

EXAMPLE.

demand the changes that may be rung on 12 bells, the different position 12 persons may sit at table ?
 479001600.

Note, This sign (\times) signifies that all numbers between which it stands are to be multiplied continually one into the other, thus : take the 12 bells, and multiply the *first* by the *second*, you have two changes on two bells ; multiply this by the third bell, you have 6 on three bells ; multiply this product by 4, or *fourth* bell, you have 24 changes, &c. as follow.

thus, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12$ will produce 479001600 changes or different positions, which you may prove very easily at leisure by *Multi-
 plication* only.*

Notwithstanding the changes on 8 bells may be out in a few hours, yet the changes on 12 (allowing 60 changes in a minute) will take 91 years, 49 days ; add to the incredibility of this increase two bells, viz. 14, would take up 16575 years to ring the changes through. See Mr. Ward, p. 85.

I thank you for giving me a short hint of these things, which I perceive the nature of very plainly : now, pray, what will you instruct me in next ?

There are two or three rules, such as the *Rule of False, Allegation, Alternate, Composition of Men*, which are not material in business, and therefore want of room must be omitted, and the more necessary part, viz. *Vulgar Fractions*, be treated of instead. And here I beg you would take care and yourself master of *Vulgar Fractions*, they being necessary in almost every branch of life, and the foundation of *Decimals*.

DIA.

EXAM

D I A L O G U E XII.

S E C T I O N I.

Of Notation of VULGAR FRACTIONS.

Tyro. WHAT do you mean by *Notation*?

Philo. *Notation*, like *Numeration*, shows you how to note, write down, or express any Fraction.

Tyro. What is a *Vulgar Fraction*?

Philo. It is a broken number or a part or parts of an integer or whole number, and consists of two parts, viz. the *Numerator*, which always stands a-top, and the *Denominator*, which always stands under it, or below it. Thus, $\frac{4}{5}$, $\frac{1}{4}$, $\frac{11}{12}$. Here 4, 3, and 11, are the *Numerators*, and 5, 4, and 12, the *Denominators*.

Tyro. How many sorts of *Vulgar Fractions* are there?

Philo. Three, viz. *First*, *Simple*, *Single*, or *Proper Fractions*, (for these are all one.) *Secondly*, *Improper Fractions*, and *thirdly*, *Compound Fractions*.

Tyro. What is a *Simple Fraction*, or how is it known?

Philo. *Simple Fractions* have their *Numerators* less than their respective *Denominators*. Thus, $\frac{1}{2}$, $\frac{3}{4}$, &c. are all *Simple Fractions*.

Tyro. What is an *Improper Fraction*?

Philo. *Improper Fractions* have their *Numerators* greater than their *Denominators*. Thus, $\frac{4}{3}$, $\frac{9}{5}$, $\frac{87}{14}$, $1\frac{2}{3}$, &c. are all *Improper Fractions*.

Philo. *Compound Fractions* are *Fractions of Fractions*, compounded or joined together by the word *of*. Thus, $\frac{2}{3}$ of $\frac{3}{5}$, or $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{12}$ are *Compound Fractions*, thus read: 2 thirds of 3 fifths, or 3 fourths of 5 sixths of 11 twelfths of an integer or whole number.

Q. I understand you, Sir; but this is very difficult
surely.

A. You have nothing to do with this at present;
I will know it by and by.

Q. What do you mean by a *mixt number*?

A. A *mixt number* is a *whole number*, with a
fraction after it. Thus, $14\frac{2}{3}$, and $147\frac{5}{8}$, are *mixt num-*

bers. I understand you; but how am I to find the
value of different *fractions*?

A. By *Reduction*, which is always learned first,
before you can't *add, subtract, multiply, or divide*, 'till
fractions are first reduced to their proper order: but
if you proceed any further, it will be proper to
learn the following signs or characters by heart, so as
to know what they signify, which will be a great help

Of the Signs used in Vulgar Fractions.

The sign (=) is the sign of Equality, and signifies that
the numbers before it are equal to those after it. Thus,
 $2=7$ is thus read, 2 and 7 is *equal* to 7.

The mark (+) is the sign of Addition, and signifies
that the numbers are to be added together. Thus,
 $2+7=15$: That is, 2 *more*, 7 *more*, or 2
added to 7 is *equal* to 15.

The sign (-) is the sign of *subtraction*, and signifies the
number after it is taken, or is to be taken out of the
number before it. Thus, $15-7=8$, that is, 7 taken
from 15, is *equal* to 8.

The sign (x) is the sign of continual Multiplication.

The sign (÷) is the sign of *Division*, and signifies the
number before it is to be divided by the number that
follows it. Thus, $56\div 8=7$, that is, 56 divided by 8
is *equal* to 7.

SECTION II.

REDUCTION OF VULGAR FRACTIONS

CASE I.

To reduce a Mixt Number to an Improper Fraction

MULTIPLY the whole number by the denominator of the fraction, and take in the numerator besides; then place the denominator under the product and it is done.

Reduce $57\frac{3}{5}$ to an improper fraction.

$$\begin{array}{r} 57\frac{3}{5} \\ 5 \\ \hline 288 \\ \hline \end{array} \quad \text{Ans. } 288\frac{3}{5}$$

Reduce $1753\frac{1}{2}$ to an improper fraction. *Ans.* $3507\frac{1}{2}$

CASE 2.

To reduce an Improper Fraction to a Whole or Mixt Number.

Divide the numerator by the denominator, and what remains, place it for a new numerator over the denominator.

Reduce $288\frac{3}{5}$ to a mixt number.

$$\begin{array}{r} 5 \overline{)288} \\ \hline 57\frac{3}{5} \end{array} \quad \text{Ans.}$$

Reduce $3507\frac{1}{2}$ to a mixt number. *Ans.* $7014\frac{1}{2}$

☞ This is only a proof to the foregoing case, the same question backwards.

C A S E 3.

Reduce or make a *Whole Number into an Improper Fraction.*

Multiply the whole number by any figure at pleasure, place the product over the figure you multiplied and you have an *improper fraction* equal to the whole number.

Reduce 14 to an *improper fraction*, whose denominator must be 9. *Ans.* $12\frac{6}{9}$.

Here, because it says 9 must be the denominator, I multiply 14 by 9, and it is 126, which I place over the 9 and it is done. *Ans.* $12\frac{6}{9}$.

N O T E 1.

To express any *whole number* fraction-wise, it is only unity or 1 under it. Thus, 14, 26, 490, &c. be $1\frac{4}{1}$, $2\frac{6}{1}$, $490\frac{0}{1}$, &c. Remember this, for it is very

N O T E 2.

Every *improper fraction* is more than an unit or 1, every *simple fraction* is less than unity or 1. Thus, *simple fraction* $\frac{3}{4}$ of a £. sterling is but 15 shillings: of a £. sterling is £. 1, and $\frac{1}{3}$ over; for if you divide 4 by 3, it is 1 and $\frac{1}{3}$, that is, £. 1-6-8.

N O T E 3.

When the numerator and denominator are alike, the fraction is equal to a whole number. Thus, $\frac{4}{4}$ is 1, or equal to 1; because the numerator divided by the denominator produces 1.

C A S E

C A S E 4.

To reduce a Compound Fraction to a Simple of the same Value.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Note, *N. N.* signifies *new numerator*, and *N. D.* *new denominator*, and *C. D.* *common denominator*, which remember.

1. Reduce $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a simple fraction? *Ans.*

For $2 \times 3 \times 5 = 30$ *N. N.* and $5 \times 4 \times 8 = 160$ *N. D.*

That is, 2 multiplied by 3, and that product is equal to 30 *N. N.*; and 5 multiplied by 4, and product by 8, is equal to 160 *N. D.* *Ans.* $\frac{30}{160}$.

*** *Note*, When there are cyphers in the numerator and denominator, then cut them off, and the fraction still the same. Thus, in the foregoing answer fraction is $\frac{30}{160}$, viz. $\frac{3}{16}$. So $\frac{4000}{7000}$ is only $\frac{4}{7}$.

2. Reduce $\frac{3}{5}$ of $\frac{5}{8}$ of $\frac{7}{9}$ of $\frac{11}{12}$ to a simple fraction? *Ans.* $\frac{11}{24}$.

C A S E 5.

To reduce a Fraction to its lowest Term, that is equal to the original given Fraction.

Divide the numerator and denominator by any number that will divide them without any remainder; and the last quotient be a new numerator, and a new denominator equal to the given fraction.

1. Reduce $\frac{72}{160}$ into its lowest terms? *Ans.* $\frac{9}{20}$.

divisors, $\frac{2}{120} | \frac{6}{80} | \frac{2}{10} | \frac{3}{5}$ *Ans.* $\frac{3}{5} = \frac{72}{120}$. So also $\frac{168}{448}$ is $= \frac{3}{8}$, which you may prove by dividing the numerator and denominator by any figure that will divide them both, as said before.

Note. There is another way to reduce a fraction to lowest terms at once, and that is by finding a common measure.

To find a Common Measure.

Divide the denominator by the numerator, and if anything remains, divide the last divisor by such remainder, and if any thing again remains, divide the divisor thereby: thus go on till nothing remains, your last divisor is the *common measure*, or number that will divide the numerator and denominator at once into their lowest terms.

Reduce $\frac{168}{448}$ into its lowest terms. *Ans.* $\frac{3}{8}$.

$$\begin{array}{r} 168 \overline{) 448} (2 \\ \underline{336} \\ 112 \overline{) 168} (1 \\ \underline{112} \\ 56 \overline{) 112} (2 \\ \underline{112} \\ 0 \end{array}$$

Here I divide the denominator 448, by the numerator 168, and 112 remains; then I divide 168, the last divisor, by that 112, and 56 remains; *lastly*, I divide 112 by 56, and 0 remains: so is 56 the *common measure*, that will divide both at one work.

$$\begin{array}{r} 168 \overline{) 448} (3 \text{ N. numerator} \\ \underline{504} \\ 168 \end{array} \quad \begin{array}{r} 56 \overline{) 448} (8 \text{ N. denominator} \\ \underline{448} \\ 0 \end{array} \quad \text{Ans. } \frac{3}{8} = \frac{168}{448}$$

Ans. $\frac{3}{8}$. Reduce $\frac{512}{1280}$ to its lowest terms. *Ans.* $\frac{2}{5}$.

C A S E 6.

To reduce Fractions of different Denominators to Fractions of the same Value, having one common Denominator.

First, Multiply all the denominators together, for a common denominator; then begin with the first numerator, and multiply it into all the denominators except its own denominator: do thus with all the other numerators, multiplying them into all the denominators except their own; so will these products be new numerators, which must be placed over the common denominator, and each fraction will be equal to the fraction whose numerator you multiplied into the denominators.

1. Reduce $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$ to fractions, having one common denominator.

First, I multiply all the denominators together, it makes 120 for a common denominator. That is, $3 \times 5 \times 8 = 120$ C. D. Then I begin with the numerator 2, that is, $2 \times 5 \times 8 = 80$ N. N. Then the numerator 3, $3 \times 3 \times 8 = 72$ N. N. And lastly, $5 \times 5 \times 3 = 75$ N. N. These new numerators I place over the common denominator, and the work is done. Thus $\frac{80}{120} = \frac{2}{3}$, for 80 is $\frac{2}{3}$ of 120. Also, $\frac{72}{120} = \frac{3}{5}$, and $\frac{75}{120} = \frac{5}{8}$.

2. Reduce $\frac{1}{2}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{4}{5}$ to fractions having a common denominator. Ans. $\frac{720}{960} = \frac{1}{2}$, $\frac{800}{960} = \frac{5}{6}$, $\frac{960}{960} = \frac{7}{8}$, $\frac{768}{960} = \frac{4}{5}$.

C A S E 7.

To reduce Fractions of one Name or Denomination to another.

I. ASCENDING.

When a fraction is to be reduced from a less to a greater denomination, then make the given fraction

to a compound *fraction*, by considering how many the *less* make one of the *greater* denomination; then reduce this *compound fraction* to a simple one, and it is one.

1. Reduce $\frac{1}{4}$ of a penny to the fraction of a £. sterling. *Ans.* $\frac{1}{960}$.

Here, because 12 pence make 1 shilling, and 20 shillings a £. I say, $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$, which reduced to a simple *fraction* by multiplying the numerators together for a *N. N.* and all the denominators for a *N. D.* gives $\frac{1}{960}$ of a £. = $\frac{1}{4}$ of a penny.

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a guinea. *Ans.* $\frac{3}{4032}$, viz. $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{21}$ = $\frac{3}{4032}$.

3. Reduce $\frac{1}{4}$ of a lb. to the fraction of a ton. *Ans.* $\frac{1}{2560}$.

2. DESCENDING.

When the *fraction* is to be brought from a *greater* to a *less* denomination, then make of it a *compound fraction*, as before; descending from the *great* to the *lesser* denomination: then multiply every denominator by the numerator of the given *fraction* (except its own denominator) and place the product over the denominator to the said given fraction, and you have the answer.

N. B. This *case* is a proof to the last.

1. Reduce $\frac{1}{960}$ of a £. sterling to the fraction of a penny. *Ans.* $\frac{240}{960}$, or $\frac{1}{4}$. For $\frac{1}{960}$ of $\frac{1}{20}$ of $\frac{1}{12}$. Now the numerator 1 multiplied into the denominator 20, and 12 is = 240, which I place over the denominator 960, thus, $\frac{240}{960}$. *Ans.*

2. Reduce $\frac{3}{4032}$ of a guinea to the fraction of a farthing. *Ans.* $\frac{3024}{4032}$. For it is $\frac{3}{4032}$ of $\frac{1}{21}$ of $\frac{1}{12}$ of $\frac{1}{4}$. Now $3 \times 21 \times 12 \times 4 = 3024$. *N. N.*

CASE 8.

To find the Value of a Fraction in Money, Weight, Measure.

Multiply the numerator of the fraction by the nearest part belonging to the integer itself, and divide by the denominator; then multiply the remainder the next nearest part of the integer, and divide by the same divisor or denominator, and thus go on 'till you have gone through all the parts of the integer, and the quotients will be your answer.

1. What is the value of $\frac{26}{30}$ of a £. sterling? *Ans.* 17s. 4d.

$$\begin{array}{r} 26 \text{ numerator} \\ 20 \\ \hline 3 \overline{) 10} 52 \overline{) 0} \end{array}$$

$$\begin{array}{r} 17s. 10s. \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \overline{) 10} 12 \overline{) 0} \\ \hline 4d. \end{array}$$

Here I multiply the numerator 26 by 20, the pence in a £. and divide by the denominator 30, 17s. and 10 over, which I multiply by 12, and divide again by 30, gives 4 pence; therefore I find $\frac{26}{30}$ of a £. to be 17s. 4d.

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2. What is the $\frac{15}{27}$ of a moidore? *Ans.* 1s. $10\frac{1}{2}d$.
by the following work.

15 numerator
27 multiply

$$\begin{array}{r} 216 \overline{) 405} \text{ (1s.} \\ 216 \end{array}$$

$$\begin{array}{r} 189 \\ 12 \end{array}$$

$$\begin{array}{r} 216 \overline{) 2268} \text{ (10d.} \\ 216 \end{array}$$

$$\begin{array}{r} 108 \\ 4 \end{array}$$

$$\begin{array}{r} 216 \overline{) 432} \text{ (2 qrs.} \\ 0 \end{array}$$

Ans. 1s. $10\frac{1}{2}d$.

Here I multiply the numerator 15 by 27, the shillings a moidore, and divide by the denominator, gives 1 shilling, and 189 over, which I multiply by 12, and divide by 216, as before, and so I go on, as appears in the work, and find $\frac{15}{27}$ of a moidore to be 1s. $10\frac{1}{2}d$.
Tyro. I see the nature of it plainly; and like this case very well; for it is diverting, as well as improving.
Philo. You are to proceed the same in weight and measure; therefore I shall only set you a few questions for practice to try at leisure.

3. What is the $\frac{324}{972}$ of a ton? *Ans.* 6 C. 2 qrs. 18 lbs.
or. 10 dr. $\frac{648}{972}$.

4. What is the $\frac{14}{112}$ of a barrel? *Ans.* $4\frac{1}{2}$ gallons.

5. What is the $\frac{152}{225}$ of a mile? *Ans.* 1173 yards, 1

6. What is the $\frac{14}{8}$ of an acre? *Ans.* 3 rods, 2 rods.

7. What is the $\frac{1}{12}$ of a year (allowing 365 days)? *Ans.* 152 days, 2 hours.

SECTION III.

ADDITION OF VULGAR FRACTIONS.

Tyro. **H**OW are *Vulgar Fractions* added together?

Philo. Very easily, by this one rule, viz. All *compound fractions* must be reduced to *simple ones* and all fractions of different denominations to a *common denominator*; then add all the numerators together, and place their sum over the *common denominator*, is the answer; and if it be an *improper fraction* at last, then reduce it to a *mixt number*, and the work is done.

1. Add $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, and $\frac{4}{11}$ together? *Ans.* $\frac{10}{11}$.

Here being one *common denominator* to all the fractions, I only add the numerators together, and the work is done. *Ans.* $\frac{10}{11}$.

2. Add $\frac{25}{27}$, $\frac{14}{27}$, and $\frac{19}{27}$ together. *Ans.* $\frac{58}{27} = 2 \frac{4}{27}$.

Here I find the sum of all the numerators to be 58, which I place over the denominator 27; but being an *improper fraction*, I divide 58 by 27, and have for answer 2 and $\frac{4}{27}$.

3. Add $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$ together. *Ans.* $\frac{227}{120}$.

Here the *fractions* having different denominators reduce them (by *case 5*) to a common denominator, and find them to be $\frac{80}{120}$, $\frac{72}{120}$, and $\frac{75}{120}$. Then I add the numerators together, and find them 227: so the answer is $\frac{227}{120} = 1 \frac{107}{120}$.

4. Add $\frac{2}{3}$ of $\frac{1}{5}$ and $\frac{1}{2}$ of $\frac{3}{8}$ together. *Ans.* $\frac{141}{240}$.

First, $\frac{2}{3}$ of $\frac{1}{5}$ is $= \frac{2}{15}$, and $\frac{1}{2}$ of $\frac{3}{8} = \frac{3}{16}$. Now $\frac{2}{15}$ and $\frac{3}{16}$ reduced to a *C. D.* is $\frac{4}{240}$ and $\frac{9}{240}$, which added together is $\frac{141}{240}$. *Ans.*

5. Add £.14 $\frac{3}{4}$, £.19 $\frac{2}{3}$, £.47 $\frac{5}{8}$, and £.100 together. *Ans.* £.181 $\frac{11}{24}$, or = 18s. 4d.

6. What must I add to £.54 $\frac{2}{3}$ and £.19 $\frac{3}{4}$ to make it £.100? *Ans.* £.25 $\frac{11}{12}$.

SECTION IV.

SUBTRACTION OF VULGAR FRACTIONS.

Tyro. **H**OW is *Subtraction of Fractions* performed?
Philo. By the same rule as in *Addition*, first reducing all *compound* to *simple*, and all to a *common denominator*; thus, *subtract* the numerator or the *less* fraction from the numerator of the other, and place the difference over the denominator, is the answer.

From	—	$\frac{14}{25}$	
Take	—	$\frac{11}{25}$	
Difference		$\frac{3}{25}$	<i>Ans.</i>
Proof		$\frac{14}{25}$	

From	£.67 $\frac{3}{4}$	
Take	18 $\frac{4}{7}$	
Difference	49 $\frac{17}{28}$	<i>Ans.</i>
Proof	67 $\frac{3}{4}$	

Here in both these examples I take the numerator of the *less* fraction from the numerator of the top, or *greater* fraction, and place the difference over the denominator. Then I prove the work, as in common *subtraction*, by adding the numerator of the difference to the numerator of the *less* fraction, and it gives the numerator of the top fraction. And thus for all examples of this sort.

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Tyro. I understand you very well, sir; but suppose the numerator of the fraction to be subtracted be larger than the numerator of the fraction I am to take it out of, how am I to do then?

Philo. This is easy enough; for when you cannot take the lower numerator out of the top numerator, then take it out of the common denominator, and that difference add the top numerator, and place over the denominator for the true difference; only pr remember this is called *borrowing*, as in common *Subtraction*, and you must carry 1 to the next figure for doing.

2. Lent $134\frac{1}{2}\frac{4}{4}$ is = £.134 - 11 - 8 *The Pro*
 Received $95\frac{1}{2}\frac{8}{4}$ is = 95 - 15 *by comm*

Due $38\frac{2}{2}\frac{0}{4}$ *Ans.* is = £.38 - 16 - 8 *Subtraction*

Proof $134\frac{1}{2}\frac{4}{4}$ Proof £.134 - 11 - 8

From $\frac{2}{3}$ of $\frac{3}{8}$ take $\frac{1}{2}$ of $\frac{3}{8}$. That is, from $\frac{6}{12}$ take $\frac{4}{12}$. Now $\frac{6}{12}$ is $\frac{2}{4}$, and $\frac{4}{12}$ is $\frac{1}{3}$. Then I take $\frac{4}{12}$ from $\frac{6}{12}$, and there remains $\frac{2}{12}$. *Ans.*

4. From 1 take $\frac{1}{11}$. *Ans.* $\frac{10}{11}$. For I make an *improper* fraction equal to 1, which has a denominator equal to the fraction to be subtracted, viz. $\frac{11}{11}$ is = 1. Then $\frac{1}{11}$ from $\frac{11}{11}$ remains $\frac{10}{11}$. And thus for any other example.

SECTION

SECTION V.

MULTIPLICATION

OF

VULGAR FRACTIONS.

Ques. HOW is *Multiplication* performed?

Philos. The fractions must be reduced to simple ones, as before directed, and mixt numbers to improper fractions. Then the rule is, multiply the numerators together for a new numerator, and the denominators together for a new denominator, and the work is done.

1. Multiply $\frac{4}{3}$ by $\frac{3}{4}$. *Ans.* $\frac{12}{12}$. For $4 \times 3 = 12$. N. N.
 $3 \times 4 = 12$. N. D.

2. Multiply $2\frac{1}{2}$ by $\frac{4}{9}$. *Ans.* $9\frac{8}{9} = 10\frac{8}{9}$.

3. Multiply $4\frac{1}{2}d.$ by $4\frac{1}{2}d.$ *Ans.* $20\frac{1}{4}d.$

First, I reduce $4\frac{1}{2}$ to an improper fraction, which is therefore I multiply $\frac{9}{2}$ by $\frac{9}{2}$. *Ans.* $8\frac{1}{4} = 20\frac{1}{4}$, as before.

4. Multiply $\text{£}.4\frac{3}{8}$ by $\text{£}.2$. Here $\text{£}.4\frac{3}{8} = 3\frac{5}{8}$, and $\text{£}.2$ expressed in fractions $\frac{2}{1}$; therefore multiply $3\frac{5}{8}$ by $\frac{2}{1}$.
Ans. $7\frac{10}{8} = \text{£}.8\frac{5}{4}$, viz $\text{£}.8 - 15$.

5. Multiply $\frac{2}{3}$ of $\frac{5}{8}$ by $\frac{3}{7}$ of $\frac{1}{5}$, that is, $\frac{10}{24}$ by $\frac{3}{35}$.
Ans. $\frac{30}{840} = 8\frac{3}{4}$. And thus may any sum be multiplied by another, with more exactness than by any other way, though not so easily as *Decimal Fractions*.

SECTION VI.

DIVISION OF VULGAR FRACTIONS.

Tyro. **H**OW is *Division of Fractions* performed?
Philo. First reduce all compound to simple fractions; and all mixt numbers to improper fractions as before directed; then the rule is, multiply the numerator of the dividend into the denominator of the divisor, for a new numerator, and the numerator of the divisor (or fraction you divide by) into the denominator of the dividend for a new denominator, and you have the answer.

1. Divide $\frac{8}{19}$ by $\frac{1}{12}$. *Ans.* $\frac{96}{19} = 1\frac{7}{19}$. Here I multiply 8 into 12, which is 96, for a new numerator, and 19 into 12, which is 228, for a new denominator, which is $\frac{96}{228}$. Here you see the answer is an improper fraction but if you change the fractions, that is, if $\frac{1}{12}$ were to be divided by $\frac{8}{19}$, then the answer would be $\frac{19}{96}$, which is a simple fraction.

Note. The character (\div) signifies division, or that the number is divided by what follows it.

2. Divide £.47 $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of a £. *Ans.* $42\frac{6}{11}$. For $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$, and $\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{4}$; therefore, divide £.47 $\frac{1}{2}$ by $\frac{2}{4}$, that is, divide $284\frac{1}{2}$ by $\frac{2}{4}$. First, $284 \times 15 = 4260$ N. N. and $6 \times 6 = 36$ N. D. *Ans.* $42\frac{60}{11}$, or £.118 $\frac{6}{11}$ = 6s. 8d.

Tyro. Then I perceive that *Division of Fractions* makes more of a sum after divided than before.

Philo. Yes, certainly, when the divisor is a simple fraction, or less than unity, as in this case; and it answers the same end as common *Multiplication*, viz. it increases the value.

Tyro. Explain this a little more, pray.

Philo. Observe then the following example.

3. It is required to bring 42 guineas into farthings by division only?

Here by case 4, I reduce a farthing to the fraction of a guinea, and find it $\frac{1}{160}$ for a divisor. Then I make 42 guineas a dividend, thus, $42\frac{2}{3}$. Now, $42\frac{2}{3} \div \frac{1}{160} = 42336$ farthings.

From hence, *Tyro*, arises this observation, that where any whole number is divided by a simple fraction, the quotient will be so much larger than the dividend, as the divisor is less than the unity, or 1; but, on the contrary, when a simple fraction is to be divided by a whole number, then the quotient will be so many times less than the dividend, as the divisor exceeds unity.

Thus £.4 divided by $\frac{1}{4}$ of a £. is the same as to multiply it by £.4, viz. £.16: but $\frac{1}{4}$ divided by 4 is $= \frac{1}{16}$ only, viz. but 1s. 3d.*

And now, *Tyro*, we are come to that rule wherein all the others are exercised, viz.

SECTION VII.

The RULE OF THREE IN VULGAR FRACTIONS.

Tyro. HOW is this rule performed?

Philo. After having reduced compound fractions and mixt numbers as before directed, first (as in the common *Rule of Three*) make your first and third

* See more of the nature of *Vulgar Fractions* in *Young Algebraist's Companion*, dialogue 3, case

M 5

number

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number of one name, then multiply your second by your third, and divide by your first.

Or rather thus:

Having stated the question, and reduced the fractions, multiply the denominator of the first number into the numerators of the second and third numbers for a new numerator, and the numerator of the first number into the denominator of the second and third for a new denominator, and that is your answer required.

1. If $\frac{1}{2}$ of a yard cost $\frac{1}{3}$ of a £, what cost $5\frac{1}{4}$ yards

$$\begin{array}{r} 4 \\ \hline 205 \\ \hline 4 \end{array}$$

Thus $\frac{3}{4}, \frac{5}{8}, 205$. Now, I begin with the denominator of the first fraction, thus, $4 \times 5 \times 205 = 410$ N. N. And $3 \times 6 \times 4 = 72$ N. D. So is the answer $410 \div 72 = 5\frac{1}{4}$ of a £. = £. 56 - 18 - 10 - 29. $\frac{4}{72}$ or $\frac{2}{3}$.

2. If a load of wheat cost £. 15 $\frac{2}{12}$, what cost 1 bushel?

Here, as 40 bushels make 1 load, my first number will be 40 , the second number reduced to an improper fraction is $1\frac{8}{12}$, and 1 bushel will be $\frac{1}{40}$. Then, if be $1\frac{8}{12}$ what $\frac{1}{40}$. Now $1 \times 182 \times 1 = 182$ N. N. and $40 \times 12 \times 1 = 480$ N. D. Ans. $\frac{182}{480}$, which is $= 75. 7d.$

Tyro. This is easy enough, if this be a general rule.

Philo. It is, and if you look over what has been done, you may with ease do any thing in this rule; therefore, I shall only set you a few questions for practice.

3. W

RULE OF THREE IN VULGAR FRACTIONS. 251

3. What is the interest of £.219 $\frac{2}{3}$ for one year, at 5 $\frac{3}{8}$ per cent. *Ans.* $\frac{28337}{2100}$, viz. £.11 - 16 - 1 - 2q. $\frac{1920}{2100} = \frac{4}{5}$.

4. Bought South Sea stock to the value of £.420 13s. 4d. and gave £.95 $\frac{4}{5}$ per cent. what comes it to? *Ans.* £.402 - 19 - 11 - 2q. $\frac{108}{350}$ or $\frac{18}{55}$.

5. A person left 40 shillings to 4 poor widows, A, B, C, and D. To A he left $\frac{1}{3}$, to B $\frac{1}{4}$, to C $\frac{1}{5}$, and to D $\frac{1}{6}$, desiring the whole might be distributed accordingly; I demand the proper share of each?

Take $\frac{1}{3}$, $\frac{1}{4}$, &c. of 40 shillings, and add them together, it makes but 38 shillings: then say,

If 38s. be $\frac{1}{3}$, viz. 13s. 4d. what will 40 be? Thus proceed with all their shares, you will find A must have 14s. $\frac{16}{38}$, B 10s. 6d. $\frac{12}{38}$, C 8s. 5d. $\frac{2}{38}$, and D 7s. 0d. $\frac{8}{38}$.

And now, *Tyro*, we will proceed to

D I A L O G U E XIII.

SECTION I.

NOTATION OF DECIMAL FRACTIONS.

Tyro. **W**HAT do you mean by a decimal?

Philo. Any number, whether cyphers before it or not, having a dot before it, thus, .2, .005 or .4715, &c. are decimals.

Tyro. How are these decimals formed?

Philo. Every decimal is a vulgar fraction, having as many cyphers for its denominator and an unit besides. Thus the foregoing decimals expressed in vulgar fractions are $\frac{2}{10}$, $\frac{5}{1000}$, and $\frac{4715}{10000}$, &c. but the denominators

nators are cast away in decimals, because you see they are always known, and we work only with the numerator, or the decimal itself; and this is the reason that decimal fractions are so very easy to what vulgar are.

Tyro. I understand you, but have you no mixt numbers here as well as in vulgar fractions?

Philo. Yes: a whole number and a decimal after it is a mixt number. Thus, 47.5 or 5.25 are mixt numbers, signifying 47 whole numbers, and $\frac{5}{10}$ parts or $\frac{1}{2}$; and the other is 5 whole numbers, and 25 hundredth parts, or 1 quarter, for 25 is $\frac{1}{4}$ of 100.

Note 1. Cyphers after figures are of no signification in decimals, thus, .5000 is but .5, and .750000 but .75, for .500 is $\frac{5}{1000}$, which is the same as $\frac{5}{10}$: but cyphers before decimals are necessary, for they decrease their value. Thus, .5 is 5 tenth parts, or $\frac{1}{2}$, but .05 is but 5 one hundredth parts, viz. $\frac{5}{100}$ in vulgar fractions.

Note 2. Pray remember this, that .5 signifies the half of any whole number, thing, or integer; .25 is one quarter, and .75 signifies three quarters. For 5 is $\frac{1}{2}$ of 10, 25 is $\frac{1}{4}$ of 100, and 75 is $\frac{3}{4}$ of 100.

This being well understood, you may proceed to *Addition*.

SECTION II.

ADDITION OF DECIMALS.

Tyro. **H**OW is *Addition of Decimals* performed?

Philo. The same as common *Addition*, only with this difference, that you must set tenths under tenths, and hundred parts under hundred parts, not regarding the number of decimal places, but set them all even towards the first left-hand row that stands

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ands next the whole numbers, and then add them all together as in whole numbers.

One example well explained will be sufficient, for you will see the reason more plainly in *Reduction*.

1. Add £. 47.45 .7259 5.25 .000545 and 15.5 together.

$$\begin{array}{r} 47.45 \\ .7259 \\ 5.25 \\ .000545 \\ 15.5 \\ \hline \end{array}$$

Ans. 68.926445

Here you see I place the whole numbers under one another, as in *Addition*, and the decimals I place even next the whole numbers, not regarding how far they extend to the right-hand: then I add them as they are, like whole numbers.

2. Add .575 .005 .0005 .95 .3 and .675 together. *Ans.* 2.5055. If you set these under one another, and add them as before, you will have 2 for a whole number, and .5055 for a decimal.

Note. Always remember to part the decimals from the whole numbers by a dot. Thus, I find in casting this last sum it amounts to five figures; but as there are but four figures in the largest decimal, therefore I make a dot between the fourth and fifth figure, always keeping an equal number of decimal places.

SECTION

SECTION III.

SUBTRACTION OF DECIMALS.

Tyro. IF *Addition of Decimals* be performed in whole numbers, I imagine *Subtraction* is the same.

Philo. It is so; only place the figures to the left-hand always under one another, let them be cyphers or figures; but don't regard cyphers to the right. An example will make it quite easy.

From 47.0075
Take 15.51

Ans. 31.4975

Proof 47.0075

From .7
Take .275

Ans. .425

Proof .7

From 5.25
Take .9840

Ans. 4.2659

Proof 5.255

Here you see I do by tens, as in whole numbers and borrow and carry the same, only I never set down cyphers to the right-hand of the figures, but entirely discard them.

2. Borrowed 100 guineas, and paid at three several times each £.27.275, what is to pay? *Ans.* £.23 - 3 - 6.

SECTION IV.

MULTIPLICATION OF DECIMALS.

Tyro. HOW is *Multiplication* performed?

Philo. Like common *Multiplication*, have no regard at all to the decimals 'till the work is done and added up: then count how many decimal places have both in the multiplicand and multiplier, and many decimals as you find in both these, so many

MULTIPLICATION OF DECIMAL FRACTIONS. 255

ures you are to prick, or point off from the right-hand to the left: so will the figures on the left hand of the dot be whole numbers, and those towards the left, the decimal parts.

EXAMPLES.

Multiply
by

7.256
.762

14512
43536
50792

Ans. 5.529072

Multiply
by

.5759
.0375

28795
40313
17277

Ans. .02159625*

Here, in example 1, I have six decimals in the multiplicand and multiplier; therefore I prick or dot off towards the left-hand, and have 5 for a whole number, and the rest are decimals.

* *Note*, In example 2, I have eight decimals in the multiplicand and multiplier; I find upon casting them up, that there are but seven places of decimals, therefore I place a cypher before the first figure, and make dot before the cypher, so I have eight decimal places.

This is a standing rule, for had the product been less in the number by 2, 3, or more places, so many cyphers must have been added to supply the deficiency.

Tyro. This is easy enough, as you observed, to that *Vulgar Fractions* are; and I perceive that money, weight, or measure, may be easily multiplied by this rule.

Philo. You say right; for suppose I was to multiply 4 - 15s. by £.3 - 10s. it would produce £.16 - 12 - 6. or £.4 - 15s. is 4.75 (because 15s. is $\frac{3}{4}$ of a £.) and 3 - 10s. is 3.5. Therefore £.4.75 \times 3.5 = £.16.625, 12s. 6d.

3. Mul-

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3. Multiply .000075 by .0015. *Ans.* .0000001125.

4. Multiply $4\frac{1}{2}$, viz. 4.5, by 4.5. *Ans.* 20.25, $20\frac{1}{4}$.

5. Multiply $\frac{1}{2}$ a crown by $\frac{1}{2}$ a crown, at a shilling the integer. *Ans.* 6.25, viz. 6s. 3d.

See *Multiplication of Vulgar Fractions*; and more the nature of this, with the way and manner of valuing any decimal, in *Reduction of Decimals*.

SECTION V.

DIVISION OF DECIMALS.

Tyro. **H**OW is *Division* performed?

Philo. The same as common *Division*; but being a little more difficult than the other rules, you must be the more careful; but if you observe the three following notes, they will help you to work at sum.

Note 1. When there are more decimal places in the dividend than in the divisor, then (after the work is done) prick off as many of the quotient to supply the defect; that is, the decimal places of the divisor and the quotient will always be equal in number to those in the dividend.

Note 2. If there be more decimal places in the divisor than in the dividend (before you begin to work) add as many cyphers to the dividend as make the number of decimal places equal to the divisor, and proceed as before directed.

Note 3. When it happens that the decimal places in the divisor and quotient (after the work is done) are not so many as those of the dividend; then you must place

DIVISION OF DECIMAL FRACTIONS. 257

place as many cyphers before the quotient, or answer, will make up that deficiency.

$$.5)47.3476$$

Ans. 946.952
See Note 1.

$$.12)63.00$$

Ans. 525.
See Note 2.

$$.12).063$$

Ans. .00525
See Note 3.

Remember that all remainders in decimals after division are of no signification, and therefore are taken no notice of.

The same is to be observed in long division as in the three foregoing examples in relation of pointing off.

$$47.15)3.747565(.0794. \text{ Ans.}$$

33005

44706

42435

22715

18860

But if thus,

$$.4715)3.747565(7.94: \text{ Ans.}$$

Here you see the same sum is proposed, but the answer is different, as you may prove at large; for the figures will be the same. And thus you see, let them be whole or mixt numbers, the work is the same.

Tyro. I understand it: and I perceive it is very easy to divide money into any number of parts by this rule.

Philo. Very easy indeed.

6. Divide £.42 - 19s. 6d. by £.4 - 15s. 6d. That is, divide £.42.975 by £.4.775. *Ans.*

7. Divide £.4.5 (viz. £.4 - 10s.) by £.3.5 (viz. £.3 - 10s.) *Ans.* £.1.285, or £.1 - 5s. 8½d.

8. Divide 1.753 by £.1753. *Ans.* .001.

This

This example, though it appears trifling at first, may be of service. For suppose it was required to divide £.1.753 among 1753 persons, the answer would be .001, viz. near 1 farthing each. Thus you may vary or alter decimals at pleasure; but as for money, and the manner of finding its value in any Decimal Fraction, that you will see in the next section, which, you observe well, you will see the whole order of decimals, and their relation and harmony, compared with *Vulgar Fractions*.

SECTION VI.

REDUCTION OF DECIMALS.

Tyro. **W**HAT does *Reduction of Decimals* teach? *Philo.* It teaches to reduce a vulgar fraction to a decimal of the same value, and also shows you how to find that value either in money, weight, measure, &c.

CASE I.

To reduce a Vulgar Fraction to a Decimal.

Add as many cyphers as you please to the numerator (making a dot between the numerator and the cyphers) then divide by the denominator as in common *Division*, and making a dot before the figures in the quotient, you have the decimal equal to the vulgar fraction.

1. Reduce $\frac{3}{4}$ to a decimal. *Ans.* .75.

$$\begin{array}{r} 4 \overline{) 3.00} \\ \underline{4} \\ 0 \\ \underline{0} \\ 00 \\ \underline{00} \\ 00 \end{array}$$

.75

Here you see .75 is $=\frac{3}{4}$, for 75 is $\frac{3}{4}$ of 100, its product per denominator.

2. Reduce

REDUCTION OF DECIMAL FRACTIONS. 259

1. Reduce $\frac{5}{8}$ to a decimal of the same value.

$$\begin{array}{r} 8 \overline{) 5.000} \\ \end{array}$$

.625 *Ans.*

That is, 625 is $\frac{5}{8}$ of 1000, its denominator.

3. Reduce $\frac{5}{393}$ to a decimal. *Ans.* .01265.

4. Reduce $\frac{3}{1753}$ to a decimal. *Ans.* .00171.

Note, Sometimes in reducing a decimal there will be remainder; but never regard that, for if you have five places in a decimal it is sufficient.

C A S E 2.

Reduce the known Parts of Money, Weight, or Measure, to a Decimal.

Add cyphers to the lowest denomination (making 2 between the cyphers and the figure) and divide by the parts contained in the next higher denomination; then place the next higher denomination before that quotient (with a dot between) and divide by the parts contained in the next higher denomination, and so go on, and your last quotient will be the decimal required. This must be illustrated by an example, to make it plain.

1. What is the decimal of 14s. 6 $\frac{3}{4}$ d.

$$\begin{array}{r} 4 \overline{) 3.00000} \\ \end{array}$$

$$\begin{array}{r} 12 \overline{) 6.75000} \\ \end{array}$$

$$\begin{array}{r} 210 \overline{) 14.56250} \\ \end{array}$$

.728125 *Ans.*

Here, according to the rule, I add cyphers to the lowest denomination, and divide by 4; then I place 6 pence before that quotient, and divide by 12. Next, I place 14 shillings before this last quotient, and

260 REDUCTION OF DECIMAL FRACTIONS.

and divide by 20, (viz. by 2) without cutting any figure or cypher off to the right-hand; for there is occasion for that.

See the proof of this in example 1, next case.

2. What is the decimal of 18s. 9½d. *Ans.* .9395

Note, If you would know the decimal of any number of shillings, from 1 to 19, observe this general rule. If the shillings be even, take the half of them is the decimal. Thus, the decimal of 16s. is .8, and of 12s. is .6, &c. But if the shillings be odd, multiply them by 5, gives the decimal. Thus the decimal of 5s. is .25; for 5 shillings is ¼ of a £. and 25 is ¼ of 100. So also the decimal of 17s. is .85, and 11s. is .55, the decimal of 1s. is .05; for there must be two places when the shillings are odd.

A RULE to find the Decimal of Shillings, Pence, and Farthings, at a £. Sterling the Integer, at once.

1. For farthings. Add cyphers to the shillings, and divide by 20.

2. For the pence. Add cyphers to the given pence and divide by 240, the pence in a £. pricking off according to the rule of *Division*. Thus you will find the decimal of 6d. .025, and of 3d. .0125.

3. For farthings. Add cyphers, and divide by 960, thus, the decimal of 3 farthings is .003125.

Note, The same is to be observed in finding the decimal of weight and measure, by adding cyphers to the given denomination, and dividing by the parts contained in the integer.

To tell the Decimal of Shillings, Pence, and Farthings by Inspection.

Note, If the shillings be even, take the half of them which will be the first decimal figure; then bring the pence and farthings into farthings, and if they be

in 25, join them to the first decimal figure, so have a decimal of three places; but if the farthings be more than 25, set down one more than they really are; if they be above 40, set down two more than they amount to; so you have the decimal nearly.

Let it be required to find the decimal of 14s. $6\frac{3}{4}d.$ and 18s. $9\frac{1}{2}d.$ as before. Here I say, the $\frac{1}{2}$ of 14 is 7, the first figure; then $6\frac{3}{4}d.$ is 27 farthings, but being above 5, I set down 28 farthings by the side of the 7, is the decimal .728, as before; and so for the other.

What is the decimal of 6s. $10\frac{1}{2}d.$? *Ans.* .344.

Here the half of 6s. is 3, and $10\frac{1}{2}d.$ is 42 farthings; being above 40, I, by the rule, set down 2 more, 44; so is the decimal .344 nearly.

Note. When the shillings are odd, multiply them by 2 and bring the pence and farthings into farthings, as before, and set the first figure under the second figure of the decimal belonging to the shillings, enlarging them by 1, or 2, as before, you have the decimal.

What is the decimal of 17s. $6\frac{3}{4}d.$ and 11s. $10\frac{3}{4}d.$?

17 multiplied by 5 is 85	$11 \times 5 = .55$
$\frac{3}{4}$ more 1 farthing 28	$10\frac{3}{4}d \times 4 + 2 = .045$
<hr/>	<hr/>
<i>Ans.</i> .878	11s. $10\frac{3}{4}d. = .595$

The next case is a proof to this, and more useful.

C A S E 3.

find the Value of a Decimal in Money, Weight, and Measure.

Multiply the decimal by the parts contained in the Integer, and prick off as many figures as there are places in the given decimal, and the figures towards the left-hand will be whole numbers, and those that are pricked off are decimals, which decimals only must be multiplied by the next denomination; thus

go

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go on, multiplying and pricking off the same number of decimals; so will the figures towards the left-hand be the value required.

1. What is the .728125 of a £. sterling?

$$\begin{array}{r} \text{s. } 14.562500 \\ \underline{12} \end{array}$$

$$\begin{array}{r} \text{d. } 6.7500 \\ \underline{4} \end{array}$$

Ans. 14s. 6½d. See Ex. of last case.

2. What is the .7615 of a guinea?

$$\begin{array}{r} 7615 \\ \underline{21} \\ 15230 \end{array}$$

$$\begin{array}{r} \text{s. } 15.9915 \\ \underline{12} \end{array}$$

$$\begin{array}{r} \text{d. } 11.8980 \\ \underline{4} \end{array}$$

Ans. 15s. 11¾d.

3. What is the .1756 of a ton?

$$\begin{array}{r} \text{C. } 3.5120 \\ \underline{4} \end{array}$$

$$\begin{array}{r} \text{qrs. } 2.0480 \\ \underline{28} \end{array}$$

$$\begin{array}{r} 3840 \\ \underline{0960} \end{array}$$

$$\begin{array}{r} \text{lb. } 1.3440 \\ \underline{16} \end{array}$$

Ans. 3 C. 2 qrs. 1 lb. 5½oz

$$\text{oz. } 5.5040$$

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Ans. 3

tell the Farthing Integer.

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Note 2. ft-hand the first fi the sec d one f mains f ext figu d if ab ate 1; en they

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2. Wha

Here I ove 5,

4. W

4. What is the .09715 of a barrel?

$$\begin{array}{r}
 36 \\
 \hline
 58290 \\
 29145 \\
 \hline
 \text{gallons } 3.49740 \\
 8 \\
 \hline
 \text{pints } 3.97920
 \end{array}$$

Ans. 3 gall. 3 pints, and $\frac{9}{10}$, or very near $3\frac{1}{2}$ gallons.

to tell the Value of any Decimal in Shillings, Pence, and Farthings. (by Inspection only) at a £. Sterling the Integer.

Note. If there be ever so many places in the decimal, the first three figures are sufficient, and all that are required in business.

Note 2. Double the first decimal place towards the left-hand, and if the second figure be under 5, then the first figure only doubled will be the shillings; but if the second figure be 5, or above 5, then you must add one shilling more to those you doubled; and what remains from the second figure above 5, carry to the next figure, and reckon them as so many farthings; and if above 25, account them still as farthings, only abate 1; but if above 45, then abate two farthings less when they really are, and you have the value required.

1. What is the value of .728125 of a £. Ans. 14s.

Here I double the first figure 7, which is 14, for the shillings, and then I say, 28 farthings is 7d. but I abate 1 because it is above .25; so it is 14s. 6 $\frac{3}{4}$ d.

2. What is the value of .39525 of a £? Ans. 7s. 10 $\frac{3}{4}$ d.

Here I say, twice 3 is 6, and the next figure being above 5, I count 1 more, which is 7 shillings; then there

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there is 4 remains from the 9, which I carry to the which is 45 farthings; but being above 40, I abate and call it 43 farthings, which is $10\frac{3}{4}d$.

3. What is the .0672 of a £. sterling? *Ans.* 1s. 4

Here the cyphers doubled is 0; but the second figure being 6, that is 1 shilling and 1 over, which I carry to the 7, is 17 farthings, and abating 1 farthing is 16 or 4 pence; which you may prove by multiplying the decimal by 20, 12, and 4, pricking off as before directed.

EXAMPLES for Exercise.

4. What is .8145 of a ton? *Ans.* 16 C. 1 qr. 4 7 oz. 10 dwts. .88.

5. What is the .275 of a lb. troy? *Ans.* 3 oz. 6 dwt.

6. What is the .0729 of a year, at 365 days the year? *Ans.* 26 days, 14 hours, 36 minutes, 14 seconds.

These examples are sufficient for any diligent learner; therefore I will proceed to put them all in practice.

SECTION VII.

The RULE OF THREE DIRECT

IN

DECIMAL FRACTIONS.

Tyro. **H**OW is the Rule of Three in Decimals formed?

Philo. The same as in the common Rule of Three Direct, by multiplying the second number by the third and dividing by the first.

RULE OF THREE IN DECIMAL FRACTIONS. 265

1. If $\frac{3}{4}$ of a yard cost $\frac{5}{8}$ of a £. what cost $51\frac{1}{4}$ yards; that is, if 3 qrs. cost 16s. 8d. what cost 51 yards, 1 qr?

Decimally thus,

If .75 be .8333, what is 51.25 yards?

Here multiply your second by the third, and divide by the first, you have the quotient 56.942, viz. 56 - 18s. 10d.

See example 1, *Rule of Three of Vulgar Fractions*.

2. If a bushel of wheat cost 3s. $9\frac{1}{2}d$. what cost 40 bushels, or 1 load? *Ans.* £.7-11-8.

Decimally thus,

If a bushel of wheat cost .189 8, what cost 40?

40

£.7.58320 *Ans.* £.7-11s. 8d.

Tyro. This last example is very short indeed.

Philo. It will always be so when the first number is or unity; for decimals are superior to vulgar fractions for ease and expedition; and though not always near the truth itself, yet answer every thing near enough for any business or demand. See the next section in variations.

3. If $5\frac{3}{4}$ ells cost £. $2\frac{803}{960}$ of a £. what cost $282\frac{7}{8}$ ells; that is, if $5\frac{3}{4}$ cost £.2-16s. $8\frac{1}{2}d$. what cost $282\frac{7}{8}$ ells?

Decimally thus,

If 5.25 ells cost £.2.8364, what cost 282.5 ells? *Ans.* £.152.625, viz. £.152 - 12 - 6.

4. If a ton of sweet oil cost £.24-17- $4\frac{1}{2}$, what cost 14 17 C. 2 qrs. 19 lb? *Ans.* £.370.1335, or 2s. 8d.

Now, *Tyro*, in all such sums as these, which either the *Rule of Three*, *Practice*, or *Vulgar Fractions*, will become very tedious; yet by finding the decimal of the odd weight, and the odd price, you have only a common multiplication sum.

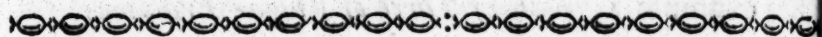
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266 RULE OF THREE IN DECIMAL FRACTIONS.

That is, multiply 14 ton .88348 by £.24 86875, you will have £. 370.1335, viz. 2s. 8d. *Anf.*

Note, You may find the decimal of 17 C. 2 qrs. 19 lb. by bringing them into lbs. and dividing by the lbs. in a ton, viz. 2240. The same for any other weight or measure, by reducing the parts, and dividing by the parts contained in the integer.



D I A L O G U E XIV.

SECTION I.

SIMPLE INTEREST.

Tyro. **H**OW is *Simple Interest in Decimals* performed?
Philo. Very easy, by the following rule.

Multiply the principal by the rate per cent. and prick off the decimals, gives the interest for one year; and if there be odd time, take the parts of the principal itself, and add to the work, it is done.

Note, For £.4 per cent. multiply by .04; for 5 per cent. by .05; for 6 per cent. .06; for 7 per cent. .07

1. What is the interest of £.147-15s. for a year, £.4 per cent. per annum? *Anf.* £. 5 - 18 - 2¼.

$$\begin{array}{r}
 \text{£. } 147.75 \\
 \underline{ .04} \\
 \text{£. } 5).9100 \\
 20 \\
 \hline
 \text{s. } 18)20 \\
 12 \\
 \hline
 \text{d. } 2)40 \\
 4 \\
 \hline
 \text{q. } 1)60
 \end{array}$$

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2. What is the interest of £.275 - 10s. for 1 year, at 6 per cent ?

$$\begin{array}{r} \text{£. } 275.5 \\ .06 \\ \hline \text{£. } 16.530 \end{array}$$

Ans. £.16 - 10 - 7 $\frac{1}{4}$

And the work will be as easy, if the per cent. be for odd time, as appears by the following examples.

3. What is the interest of £.1250 - 15s. for 3 $\frac{1}{2}$ years, at 6 per cent. per annum ?

1	£. 1250.75	.06		
1	75.0450	for 1 year.		
1	3			
1	225.1350	for three years		
1	37.5225	add for the $\frac{1}{2}$ year		
1	262.6575	for 3 $\frac{1}{2}$ years. <i>Ans.</i>		

4. A person left his wife by will $\frac{1}{3}$ of his estate, which was £.355 - 7 - 6. Now this lay in hand 3 $\frac{1}{2}$ years; I demand what interest is due upon it, at £.4 $\frac{1}{2}$ per cent. per annum ? *Ans.* £.55 - 19 - 5.

Multiply £.355.375 by £.4 $\frac{1}{2}$, that is by .045, is £.15.991875 for 1 year: this multiplied by 3.5 years, gives £.55.9715625.

And thus you see, that if the principal, interest, or time, be ever so much, yet it is easily performed by decimals.

5. A merchant made an assurance upon goods in a ship bound to a certain port, to the value of £.2530,
N 2
upon

upon condition, that in case of a total loss or damage, the assurers were to pay $\text{£} .97\frac{1}{2}$ per cent. deducting $\frac{1}{2}$ per cent. (viz. 10s. per cent.) out of it: now the ship was cast away, but there were as many goods saved as amounted to $\text{£} .955$; I demand what the merchant has to receive? *Ans.* $\text{£} .1527 - 18 - 11$.

First, $\text{£} .955$ taken from $\text{£} .2530$ remains $\text{£} .1575$ loss; this multiplied by 97.5 is 1535.625 , and deducting $\frac{1}{2}$ per cent. from this, is only cutting off two figures in the pounds, thus, $15|35$, and take the $\frac{1}{2}$ of $15|35.625$ which is 7.678125 , placing it in the units place of the pounds, and the rest in order (as follows in division) and subtract it from $\text{£} .1535.625$, leaves $\text{£} .1527.946875$ viz. 18s. 11d. *Ans.*

The proof of this is worthy your observation, Tyro. For, observe, $97\frac{1}{2}$ per cent. wants but $2\frac{1}{2}$ per cent. of $\text{£} .100$, that is, of being cent. per cent. Now if you multiply $\text{£} .1575$ by 2.5 , (viz. $2\frac{1}{2}$) it gives $\text{£} .39.375$ which added to $\text{£} .1535.625$, gives the original sum of loss, viz. $\text{£} .1575$.

Tyro. I thank you kindly, sir; for this, as you have observed, is a proof to me very plainly.

6. What comes an assurance, or a commission, for storage, or brokerage to, upon $\text{£} 3500$, at 18s. per cent.

$$\begin{array}{r} 35|00 \\ .9 \text{ the decimal of } 18s. \end{array}$$

$$\text{£} .31.5 \quad \text{Ans. } \text{£} .31 - 10.$$

And thus decimals perform any thing with ease and pleasure.

Tyro. I see it, sir; and now be pleased to give me an example or two in *Compound Interest*.

Philo. I will.

SECTION II.

COMPOUND INTEREST.

Tyro. **H**OW is *Compound Interest* performed?
Philo. By continual Multiplication and Addition.

Quest. 1. What will £. 100 amount to, if it be forborne 5 years at 5 per cent. per annum, Compound Interest. *Ans.* £. 127 - 12 - 6 $\frac{3}{4}$.

1. The common method is to multiply the principal by the per cent. and cut off two figures, (which is the same as dividing by 100) this gives the interest for one year, which is £. 5; this add to £. 100, gives £. 105 for the amount of the first year. Then this multiplied by the per cent. gives the interest for the 2d year, which added to the principal of £. 105, gives £. 110.25 for the amount of the 2d year, &c. &c. But this is tedious; therefore the best way is,

2. Set down the principal 100l. and find the interest the 1st year, and add it to it, and it makes as before 105. Set the 5 under the 100l. and make two dots after it, thus, 5.. this saves dividing by 100, and will supply the decimal places; then multiply 105l. by 5, (viz. 05 in decimals) keeping always 2 figures of the decimals under the two dots, and it produces 5—25, which, added to 105l. gives 110.25l. as before, &c. &c. See the operation.

N 3

£. 100.

COMPOUND INTEREST.

£.100 × viz. by .05 is 5.00, viz. 5
5 .. added

Amount

1 year 105 .. this × .05
is 5.25 add

2 year 110.25 this × .05
is 5.5125 add

3 year 115.7625 this × .05
is 5.7881 add

4 year 121.5506 this × .05
is 6.0775

5 year 127.6281 *Ans.* £.127.12.

Quest. 2. *A* lent *B* £.136.775, which *B* promised to pay compound interest for, at £.6 per cent. per annum and bound his heirs, executors, &c. to that condition. Now *A* died, and *B* took no notice of payment 'till it was at last discovered by the executors of *A*, and *B* had the money in hand 20 years: I demand what *B* has got to remit for the debt? *Ans.* £.438.651.

But the shortest method of all to calculate compound interest, is by the following tables:

TABLE I.

A Table shewing how much 1 Pound sterling will amount to in any Number of Years under 21, at £.6 per Cent. per Annum, Compound Interest.

Years.	£. 5 per cent.	Years.	£. 5 per cent.
1	1.05	11	1.7103
2	1.1025	12	1.7958
3	1.15763	13	1.8856
4	1.2155	14	1.9799
5	1.27628	15	2.079
6	1.3401	16	2.1828
7	1.4071	17	2.292
8	1.4744	18	2.4066
9	1.5513	19	2.5269
10	1.6289	20	2.6533

TABLE

TABLE II.

A Table shewing how much 1 Pound sterling will amount to in any Number of Years under 21, at the rate of £.6 per Cent. per Ann. Compound Interest.

Years.	£.6 per cent.	Years.	£.6 per cent.
1	1.06	11	1.8982
2	1.1236	12	2.0121
3	1.191	13	2.1329
4	1.2624	14	2.261
5	1.3382	15	2.3965
6	1.4185	16	2.5403
7	1.5036	17	2.6927
8	1.5938	18	2.8543
9	1.6894	19	3.0256
10	1.7908	20	3.2071

1. The Explanation of the Tables.

The amount of £.1 for 1 year, at £.5 per cent. is 1.05; this $\times 1.05 = 1.1025$, the amount of the 2d year; this $\times 1.05$ is 1.1576, &c. &c. The same for £.6 per cent. which is 1.06. this into 1.06 is equal to 1.1236 for the second year, &c. &c.

2. The Use of the Tables.

When any sum is given for any number of years, then multiply the given sum by the number given right against, or answering to the number of years, and you have the answer at one operation.

Let us take example 1, viz. £.100, for 5 years, at £.5 per cent.

I look in the table of 5 per cent. and against 5 years I find 1.27628 (the amount of £.1 for 5 years) this multiplied by £.100 gives 127.628, viz. £.127-12-6 $\frac{3}{4}$ as before.

Tyro. This is short indeed!

Philo. 'Tis the same as if it were for 20 years. Thus in example 2, it is required to tell the amount of £.136-15-6, for 20 years, at £.6 per cent. compound interest.

N. 4

I look

I look in the table for £.6 per cent. and against 20 years I find 3.2071, the amount of £.1 for that time; this multiplied by the sum £.136-15-6, (viz. 136.775) gives £.438.651, viz. £.436.13 shillings and 1 farthing.

Tyro. I like this very well; but suppose the years are more than in the tables, how then?

Philo. Very easy; only add any two or more numbers together, as make the number, and multiply the sums belonging to each number of years in the table together, gives the right sum for that number of years.

Thus, suppose I wanted to tell the amount of the last question for 30 years, at £.6 per cent. Here I take any two numbers, which added, make 30. Suppose for example 10 and 20; against 10 I find 1.7908, and against 20 I find 3.2071, this multiplied together gives 5.7432, the amount of £.1 for 30 years, at £.6 per cent. which multiplied into £.136.775, gives 785.526, the amount of this sum for 30 years; and thus for any other numbers.

Tyro. I heartily thank you, sir.

Philo. You see how easy it is, and this is the only method to calculate annuities, pensions, &c. except you can do it by *Algebraic* calculation.

Note, These tables are easily made; the construction depends, as I told you, in *Simple Interest*, upon this; That let the per cent. be what it will, suppose £.4 per cent. say, if £.100 be 104, what will £.1 be? *Ans.* £.1.04 for 1 year. So also for $4\frac{1}{2}$ per cent. it is £.1.045; for 5, it is £.1.05, &c.

Tyro. I understand you very well, sir, and shall endeavour, as time offers itself conveniently, to look over these things, and make myself yet more perfect.

Philo. You will perform your promise, I hope; but before I leave you, I will give you a notion of the *Extraction* of the *Square* and *Cube Roots*, being very necessary in many businesses, but especially in the art of mensuration, and several other branches of the mathematics.

DIA-

DIALOGUE XV.

SECTION I.

The EXTRACTION of the SQUARE ROOT.

Tyro. **W**HAT do you mean by the words *Square* and *Root*?

Philo. A square number is a number multiplied by itself, viz. any figure or figures multiplied by the same figure or figures, the product is the square of that number: Thus, $2 \times 2 = 4$, the square of 2; and $9 \times 9 = 81$, the square of 9.

Tyro. This is plain enough. And what is the *Root* then?

Philo. The root is that from which the square is formed: Thus, I told you before, the square of 2 is 4, and the square of 9 is 81; therefore, *vice versa*, 2 is the root of 4, and 9 is the root of 81, as appears by the following table, which should be readily known.

T A B L E.

Roots	1	2	3	4	5	6	7	8	9	10	11	12
Squares	1	4	9	16	25	36	49	64	81	100	121	144

Tyro. How is the *square Root* extracted?

Philo. I will shew you the whole process, which pray observe.

Suppose it were required to extract the *square Root* of 3136, or any other figures.

First, I set down the figures thus 3136, and beginning at the units place, I make a dot or point over it, and also over every other figure towards the left hand, as you see in the margin; and pray observe that as many dots as you have, so many figures the *Root* will always consist of, which here are 2.

Secondly, I seek (by the table) the nearest root to the figures contained in the first point of figures, viz. in

N 5

31,

31, and find it to be 5, which I place in the quotient, thus, $31\dot{3}6(5$, which figure 5 is called the root or part of the root.

Thirdly, Square the root, that is, $31\dot{3}6(5$ multiply it by itself, and place it under the said first points as in common

Division, and subtract it therefrom, 636 resolvend and bring down the next point, viz.

36, and place it by the side of the remainder, it is 636, which is called the resolvend, as in the margin.

Fourthly, Then I double the quotient figure or root 5, which is 10, and making another crooked line, I place it for a divisor right against the resolvend, thus,

Fifthly, I now ask (as in *Division*) how many times

10 I can have in the resolvend (always rejecting the last figure)

that is, how many times 10 are contained in 63, and find it 6

times, which 6 I put in the root by the side of the 5, and

also by the side of the divisor

10, which makes 106; then I multiply 106 by 6, which is 636, and nothing remains; thus I find the square root of 3136 to be 56.

P R O O F.

I square the root of 56, that is, I multiply it by itself, viz. 56 by 56, and it gives 3136.

2. What is the square root of 56169?

$56169(237$ root *Ans.*

4

43)161 resolvend

129

467)3269 new resolvend

3269

0

Here I proceed the same as in example 1, by making a dot over every other figure, and find the nearest the root of the first point 5 to be 2, which I square, and place under 5, and remains 1, to which I bring down the next two figures 61, and it is 161; then I double

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double the root 2, it is 4, which I place on the left-hand for a divisor. Then I ask how many times 4 are contained in 16, which though it be 4, yet upon trial will be but 3 times (for you must observe, it will often be less than it looks to be) which 3 I place in the quotient, and also by the side of the divisor 4, which makes it 43; then I multiply 43 by 3, and it is 129; and subtracting 129 from 161, I have 32 remains, to which I bring down the next point, or two figures 69, and it is 3269, which I call a new resolvend; then I double the root 23, which is 46 for a new divisor, and ask how many times 46 I can have in 326, and find it 7, which I place in the quotient, and also after 46, and it is 467, which multiplied by 7, gives 3269. Thus I find the *Square Root* of 56169 to be 237. And for a proof I find 237 multiplied by 237 = 56169. Do you understand it, *Tyro*?

Tyro. Very well, sir: then I perceive that after I have done with the first figure in the root, I am to double it, and take down the next point, and then double the two figures, and take down the next point, and so keep on in the same order, am I not?

Philo. Your notion is right, and if you consider well the manner of the working the last example, if you have ever so many figures, you may do it with ease.

Tyro. But suppose after the work there should be a remainder?

Philo. That matters not at all; only when you come to prove the work, after multiplying the root by itself, you must add the remainder to the product, and it will be equal to the given number, if the work be right.

Sums for Practice.

3. I demand the square root of 2996361. *Ans*. 1731.
4. I demand the square root of 3076516. *Ans*. 1754.
5. What is the square root of 43623. *Ans*. 208, and 359 remains over.

N. B. If you have a mind at any time to know what the remainder will produce, add an even number of cyphers to the sum, and double the root, and proceed as before: thus, the square root of 43623,000000 is 208,861.

2. To extract the square root of a vulgar fraction: Extract the square root of the numerator for a new numerator, and of the denominator for a new denominator.

6. What is the square root of $\frac{25}{81}$? *Ans.* $\frac{5}{9}$.

Note, When you cannot extract the root of the numerator and denominator, then reduce the vulgar fraction to a decimal, and extract the square root, you have the answer.

7. What is the square root of 1 and $\frac{3}{5}$?

First, $\frac{3}{5}$ is $=6$, which is 1.6; then I add cyphers, thus, 1.600000, and find the root to be 1.264.

Note, There must always be an equal number of decimals.

8. What is the square root of 19.2? *Ans.* 4.3817.

9. What is the square root of .0003? *Ans.* .01732.

SECTION II.

The Use of the SQUARE ROOT, applied to various Branches of the Mathematics.

10. **I** Demand what is the mean proportional number between 30 and 50. *Ans.* 38.7 tenths.

Multiply one number by the other, and add cyphers and extract the square root, you have 38.7.

11. There is a triangle, whose base is 30 inches, and the perpendicular 40; I demand the hypotenuse? *Ans.* 50.

Note, The perpendicular is that part which is right up; the base is that which lies next you, and the hypotenuse is the slanting side, called also the diagonal line.

A general Rule to find the third Side of any Triangle, having two Sides given.

1. Having the perpendicular 40, and base 30, as above, to find the hypotenuse, add the square of the base and perpendicular together, and extract the square root

root of them, you have the hypotenuse. Thus, the square of $30=900$. The square of $40=1600$, their sum is 2500 , the square root of which is 50 , the hypotenuse required.

2. Having the hypotenuse and perpendicular, or base, given, to find the other side.

From the square of the hypotenuse take the square of the base, or perpendicular, and the square root is the other side required.

EXAMPLES.

12. There is a steeple to whose vane or top a string is tied, which reaching to the ground, is in length 60 yards, and the distance from where it touches the ground to the middle of the steeple end is 25 yards; I demand the height of the steeple? *Ans.* 54.5 yards.

Square 60 is 3600 , and take the square of 25 , viz. 625 , from it, and extract the square root, you have 54.5 .

13. Suppose a ship sails from a certain port, has made 87 miles difference of latitude, and 71 miles departure, what is her distance on a regular course? This is only finding the hypotenuse; for add the square of 87 and 71 together, and extract the square root, you have 112.2 miles distance.

14. There is a circle or triangle, whose superficial content is 30800.25 , I demand the side of a square, whose superficial content shall be equal thereto? *Ans.* 175.5 .

15. Suppose a rope 5 inches round, I demand the compass of another rope, that is double the strength? Square the compass of the rope, it is 25 , which multiply by 2 , and extract the square root, it is 7.07 inches. If it were required to be 3 , 4 , 5 , or 6 times the strength, then multiply the square by 2 , 3 , 4 , 5 , or 6 , and extract the root.

16. There is a cable 10 inches round, which weighs 21 *C.* I demand the weight of one 8 inches round? *Ans.* 13.44 *C.*

As the square of the one, viz. 100, is to the square of the other, viz. 64, so is the weight of the one to the other, viz. 13.44 *C.*

17. There is a circle whose content is 153.9385, I demand its diameter.

First, As 22 is to 28::153.9385 to the square of the diameter, viz. 195.9217, whose square root is 13.99.

And thus I think I have given you sufficient examples in this rule, that you may with a little more practice become quite master of it.

Tyro. I thank you for your care, sir, and I understand it very well; I wish I understood the *Cube Rule* as well.

Philo. That you may soon do by care, and a little more pains, though it be something more difficult than this rule.

D I A L O G U E XVI.

S E C T I O N I.

The EXTRACTION of the CUBE ROOT.

Tyro. WHAT do you mean by a cube?

Philo. A cube is that which has length, breadth, and thickness: thus, suppose a piece of wood to be cut into the form of a die, which is equal every way, in length, breadth, and thickness, such a figure is called a solid, and by name a cube.

Tyro. Give me a further description of a cube in figures.

Philo.

Phil. You know any number multiplied by itself is square; so any number multiplied twice by itself is cube number; thus, the cube of 2 is 8: for 2×2 is 4, and $4 \times 2 = 8$: so also, the cube of 5 is 125: for $5 \times 5 = 25$, and $25 \times 5 = 125$. Thus you see 8 is the cube, and 2 the root of that cube; also 125 is a cube number, whose root is 5, as appears by the following table of both squares and cubes.

T A B L E.

1	2	3	4	5	6	7	8	9
1	4	9	16	25	36	49	64	81
1	8	27	64	125	216	343	512	729

Tyro. How is the cube root extracted?

Phil. To give you a rule for it (I look upon) would be too tiresome for your memory, as there are many parts contained in it: I shall therefore take an example of two, and proceed in the whole process, or order of the work.

1. I demand the cube root of 32768? *Ans* 32.

R U L E I.

First, I make a dot over every fourth figure, beginning at the units place, as in the margin, and as many dots as you have, so many places the root will contain, which here are two places, 32768.

Secondly, Seek the root, or nearest root, to the first part 32, which (by the table) is 3, and place it in the quotient, which is the first figure in the root, thus, 32768(3.

Thirdly,

Phil.

Thirdly, Cube the figure which you put in the quotient (that is, $3 \times 3 \times 3 = 27$) and place it under the first point 32, and subtract 32768(3) it therefrom, Thus, 27

5

Fourthly, To this remainder (5) bring down all the figures of the next point) viz. 32768(3) 768) and place them by the side of the remainder, and call this the resolvend, Thus, 5768 resolvend

Fifthly, Triple the quotient, (that is, always multiply it by 3) and put the units place to it under the tens place of the resolvend; and call that the triple quotient, 32768(3) 27

5768 resolvend

Thus, 9 triple quotient

Sixthly, Now square the quotient (that is 9) and triple that square, that is 27, 32768(3) 27 and place the units place of it under the tens place of the triple quotient; that is, place it one figure more to the left-hand, and call it the triple square, 5768 resolvend

9 triple quotient

Seventhly, Add these two together, and call it the divisor, 27 triple square

Thus, 279 divisor

Eight

The CUBE ROOT.

281

Eighthly, Ask how many times the divisor is contained in the resolvend, rejecting the last figure as you did in the square root; that is, ask how many times 279 you can have in 576 the resolvend, which here is 2, and place this also in the quotient, which now is

$$\begin{array}{r} 32768(32 \\ 27 \\ \hline 5768 \\ \hline 9 \text{ triple quotient} \\ 27 \text{ triple square} \\ \hline \end{array}$$

Thus, 279 divisor

Ninthly, Cube the figure last put in the quotient, viz. 2, whose cube is 8) and place the units placed under the units place of the resolvend.

$$\begin{array}{r} 32768(32 \text{ root} \\ 27 \\ \hline 5768 \text{ resolvend} \\ \hline 9 \text{ triple quotient} \\ 27 \text{ triple square} \\ \hline 279 \text{ divisor} \\ \hline 8 \text{ cube of } 2 \end{array}$$

Tenthly, Multiply the square of the figure last put in the quotient (viz. 4) into the triple quotient, (viz. $\times 9$) which is 36, and place the product under the figure more towards the left-hand.

$$\begin{array}{r} 32768(32 \text{ root} \\ 27 \\ \hline 5768 \text{ resolvend} \\ \hline 9 \text{ triple quotient} \\ 27 \text{ triple square} \\ \hline 279 \text{ divisor} \\ \hline 8 \text{ cube of } 2 \\ 36 \text{ the square of } 2 \\ \text{by trip. quot.} \end{array}$$

Eleventhly,

Eleventhly, Multiply the triple square (viz. 27) the last figure put in the quotient, and place also this, one figure more towards the left-hand, which is 54.

32768(32 root. *Ans.*
27

5768 resolvend

9 triple quotient
27 triple square

279 divisor

Twelfthly, Add these three last numbers together as they stand, and call it the subtrahend, which is 5768, equal to the resolvend.

8 cube of 2
36 sq. of 2 by tr.
54 tr. square by

5768 subtrahend

Thus is the work finished, and the cube root of 32768 is found to be 32.

P R O O F.

For the proof of this, multiply 32 by 32, and it is 1024, which 1024 I multiply by 32 again, and he 32768

Note 1. If the subtrahend had been larger than the resolvend, then I must put a less figure in the second place in the quotient, and proceed as before directed.

Note 2. When there is another point of figures to be taken down, first subtract the subtrahend from the resolvend, and to the remainder bring down the next point, calling it the new resolvend, or second resolvend; then proceed to work as after the first resolvend in every respect.

2. Another method to extract the cube root, which is in many respects easier and shorter than the former.

Let us take the last example 32768.

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First, Find the root of the first point as before, and subtract it therefrom, and to the remainder bring down the next point of figures, and call it resolvend, or dividend, which you please.

Secondly, Square the root, and multiply it by 300 for a divisor; and, as in common *Division*, see how many times it is contained in the dividend, and place it in the quotient, or root accordingly.

Thirdly, Multiply the divisor by the last figure of the root, and place it under the dividend (units under units) drawing a line between them.

Fourthly, Square the last quotient figure, and multiply it by the first quotient figure, and that product multiply by 30, and set this under the last work, units under units, &c.

Fifthly, Cube the last figure, and put the units of this under the units of the last, and add these three together, in order as they stand, which is the subtrahend; which if it be more than the resolvend, or dividend, you must put a less figure in the quotient, and proceed as before; but if it be less than the dividend, subtract it therefrom, and the work is done, for two figures in the root: But if there be more figures, bring them down to the remainder, and call it a new dividend; and square the whole root, and multiply by 300, for a new divisor, and put the figures in the quotient. Then square this last figure, and multiply it by the foregoing figures in the root, and then by 30.

Lastly, Cube the last figure, and place it as before directed, and the work is done for three places. The same to be observed for more figures.

Thus

Thus, the last example 32768(32 root. *Ans.*

$$\begin{array}{r} 27 \\ \hline \text{Divisor } 2700) \ 5768 \text{ dividend} \end{array}$$

5400

360

8

5768, subtrahend.

Tyro. I understand it very well, and think this way the easiest.

Philo. Take your choice, as I observed before; shall now only set you one sum at large, the first way which you may prove by the second.

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2. W also 234 ×

3. Wha
754.

4. I den
Ans. 21122

2. What is the cube root of 12812904? *Ans.* 234.

12812904 (234 root. *Ans.*
8

4812 resolvend

6 triple quotient
12 triple square

126 divisor

27 cube of 3
54 square of 3 by T. quotient
36 triple square by the root 3

4167 subtrahend

645904 new resolvend

69 triple quotient
1587 triple square

15939 new divisor

64 cube of 4
1104 square of 4 by T. quotient
6348 triple square by the root 4

645904 new subtrahend = new resolvend.

This you may easily prove the other way at leisure.

Also, $234 \times 234 \times 234 = 12812904$.

QUESTIONS for Practice.

3. What is the cube root of 5396209064? *Ans.*

754.

4. I demand the cube root of 9423479350146861?

Ans. 211221.

To extract the Cube Root of any Vulgar or Decimal Fractions.

5. What is the cube root of $\frac{27}{1331}$? *Ans.* $\frac{3}{11}$.

6. What is the cube root of $\frac{2197}{9261}$? *Ans.* $\frac{13}{21}$.

Extract the cube root of the numerator and denominator for a new numerator and denominator.

7. Extract the cube root of 32.768. *Ans.* 3.2.

Proceed as in whole numbers, only prick off as many decimals in the root as you have dots over the decimals.

Note, As in the square root you add either 2, 4, 6, &c. cyphers to the decimal, so here you must add by three's, that is, you must add either 3, 6, or 9, &c. cyphers.

8. What is the cube root of .002? *Ans.* .1259, and 4383021 remains.

SECTION II.

The USE of the CUBE ROOT.

9. **T**HERE is a cube whose solidity is 1372 feet. I demand the side of a cube, whose solidity four times less? *Ans.* 7.

Divide 1372 by 4, and extract the cube root.

10. If a bullet two inches diameter weighs 3 lb. what will one of the same metal weigh which is eight inches diameter? *Ans.* 192 lb.

11. Solids being in the triple proportion to their side or diameter, it is thus found :

As the cube of the given diameter is to the weight, so is the cube of the other diameter to the weight or the other required.

12. Suppose a shot of four inches diameter weighs 8 lb. I demand the diameter of another that weighs 44 lb. *Ans.* 8 inches. This is the reverse of the last question.

13. There is a sphere or globe, whose solid content is 250047 inches: I demand the side of a cube, whose solidity shall be equal to the solidity of the globe. *Ans.* 63 inches.

14. A country farmer lent his neighbour out of his hay-stack 20 feet of hay in length, breadth, and depth, and his neighbour brought it home 10 feet at one time, and ten at another, : how is the balance, and who debtor? *Ans.* 6000 feet due to him that lent it: he having received but $\frac{1}{4}$ th.

15. Suppose a ship 300 tons burthen, 75 feet by the keel, $29\frac{1}{2}$ feet by the beam, and 14 feet deep in the hold; I demand the dimensions of another ship of the same make, of 500 tons burthen?

Say, as 300 tons is to 500 tons, so is the cube of the given keel to the cube of the ship's keel required, the cube root of which is 88.9 feet, *Ans.*

And thus for the other two dimensions, which I leave for your practice.

And now, *Tyro*, before I leave you, I will give you a little hint of measuring, gauging, &c. which may possibly be of service to you, and your acquaintance. You must expect me to be very short; but you may, by your care and diligence, make a better progress.

POST-

P O S T S C R I P T.

D I A L O G U E XVII.

AS this little Treatise may fall into the hands of such persons in the country, who would be glad of an opportunity of having a notion of measuring a piece of timber, a brick wall, a cistern of malt, or common regular field, or piece of land, &c. I have (on purpose for their amusement, and instruction of those youths that have a fancy this way) added this *Postscript*, which I make no doubt will be very acceptable to all such as delight to be industriously employed at leisure times: and I persuade myself, it must be very agreeable to a parent, in either of the ways of life, to see his son diligent and ready at these things; which though he may not measure so exact for want of more learning, proper instruments, or experience, yet may come near enough the truth to give satisfaction.

Of FLOORING, ROOFING, &c.

Quest. 1. How many clinkers, 6 inches long, and 4 inches wide, will floor a stable 17 feet long, and 9 feet wide? *Ans.* 1224.

Multiply the length of the stable by the breadth gives 153 feet; this multiply by 144, the square inches in a square foot, gives 22032 inches; this divide by 18, the inches in 1 clinker, gives 1224, *Ans.*

Quest. 2. How many oak planks will floor a barn 60½ feet long, and 33½ feet wide, when the planks are 15 feet long, and 15 inches wide? *Ans.* 108.

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Multiply $60.5 \times 33.5 = 2026.75$. Then 15 feet $\times 1.25$ feet (viz. 15 inches) gives 18.75 feet for 1 plank; now $2026.75 \div 18.75$, gives 108 planks.

Quest. 3. A thatcher thatches a barn 60 feet long, and 25 feet wide, and the two porches are each 15 feet long, and 10 feet deep, I demand how many squares are contained in it? *Ans.* 33 squares. *N. B.* 100 feet is 1 square.

Multiply 60 by 25 gives 1500 feet for 1 side, which doubled, gives 3000 for both the sides; then the porch, viz. 15×10 , gives 150 for 1 side, which doubled, gives 300, which added to 3000 is 3300, which divided by 100 (that is, cutting off two figures) gives 33 squares.

2. Of PAVING, PAINTING, WAINSCOTING, &c.

Tyra. How is paving, painting, and wainscoting measured?

Philo. By the square yard; 9 square feet being one yard.

Quest. 4. A gentleman has a walk 22 yards long, and 12 feet wide, which is paved of stone; how many yards does it contain? *Ans.* 88 yards.

First, Multiply 22 yards, viz. 66 feet, by 12, gives 792, which divide by 9, gives 88 yards.

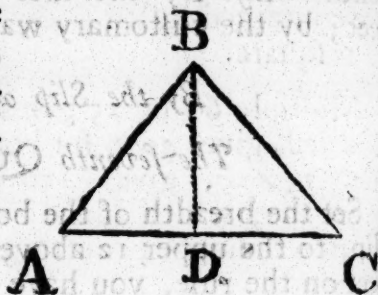
Quest. 5. There is a room 64 feet round, and 9 feet high, in which are two windows, each 6 feet high, and 3 feet wide, and the fire-place contains 9 square feet; I demand how many yards of paper, half-yard wide, will hang it? *Ans.* 118 yards.

First, $64 \times 9 = 576$ yards, the content, out of which take 18 feet each window, viz. 36 feet, and 9 the fire-place, is 45; and the remainder is 531 feet, which divide by 9, gives 59 yards, the content of the room; but as the paper is $\frac{1}{2}$ yard wide only, it will take double this number, viz. 118 yards, *Ans.*

Painting, Wainscoting, &c. are done by the yard square, and measured after this manner.

To measure the Peak End of a House, or any Triangle.

Quest. 6. Let ABC be the peak end of a roof, whose base AC measures 24 feet, and the perpendicular line BD from the top of the peak 16, I demand how many square yards it contains?



Multiply $\frac{1}{2}$ the perpendicular BD by the whole base or line AC ; or else, multiply the whole perpendicular BD by $\frac{1}{2}$ the base AC , viz. AD or CD , gives the content in feet, which divide by 100, gives the squares, or by 9, gives square yards.

Thus, AC , 24 feet, multiplied by $\frac{1}{2} BD$, 16 feet (viz. 8 feet) gives 192 feet, viz. 1 square, 92 feet: or divide by 9, gives $21\frac{2}{3}$ square yards of plaistering, and thus for any other triangle.

3. Of BOARD and TIMBER-MEASURE.

1. If the board be regular, multiply the length in inches, by the breadth in inches, and divide by 144 gives the answer; or, multiply the length in feet by the breadth in inches, and divide by 12, gives the answer.

Quest. 7. There is a board 9 feet long, and 10 inches wide; how many feet does it contain? *Ans.* $7\frac{1}{2}$ feet.

If the board be wider at one end than another, the true way is, to multiply the ends together, and extract the square root for a mean breadth. But custom has brought up an easier way, though not so true, viz. add the breadth of both ends together, and take the

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6 inches,

for a mean breadth, then multiply the length by this mean breadth into inches (as before directed) and divide by 144.

Quest. 8. There is a board 16 inches at one end, and 8 at the other, and 10 feet long; what is the content? *Ans.* By the first true way it is very near $9\frac{1}{2}$ feet; by the customary way, it is 10 feet.

By the Slip or Sliding Rule.

The seventh QUESTION proved.

Set the breadth of the board, viz. 10 inches, on the slip, to the upper 12 above on the rule; then against 9 feet on the rule, you have $7\frac{1}{2}$ feet on the slip, the content required.

Again, Suppose a board be 14 feet long, and 15 inches wide, what is the content? Set 15 on the slip, against the upper 12, then, against 14 feet on the rule, you have $17\frac{1}{2}$ feet on the slip, answer.

Tyro. I understand you, Sir; but pray how do you measure timber?

Philo. The customary way is this: with a small string or cord, take the circumference of the tree (which is done in any place, where the buyer and seller can agree) then double this string into four parts, and apply it to your rule, and that length is called the girt, or $\frac{1}{4}$ part of the circumference; and it is customary to abate one inch of the girt on account of the bark.

2. Having got the girt, multiply it by itself, that is, square it, and multiply that product by the length of the tree in feet, and divide by 144, gives the content; or multiply it by the length in inches, and divide by 1728, gives the content.

Note, Few persons mind less than $\frac{1}{2}$ a foot in the length of a tree, except it be very large.

Quest. 9. There is a tree 14 inches girt, and 9 feet long, I demand the content? *Ans.* $12\frac{1}{4}$ feet.

First, 14, multiplied by 14, is 196, this $\times 9$, the length, = 1764, which divided by 144, gives 12 feet, 6 inches, which is $\frac{1}{4}$ of 144, viz. $12\frac{1}{4}$ feet content.

Quest. 10. There is a tree $10\frac{1}{2}$ inches girt, and $12\frac{1}{2}$ feet long, I demand the content? *Ans.* $9\frac{1}{2}$ feet. For $10.5 \times 10.5 = 110.25 \times 12.5 = 1376.125 \div 144 = 9.57$ feet, or 9 feet, 82 inches, *Ans.*

By the Slip or Sliding Rule.

The ninth QUESTION proved.

Set the length of the tree on the slip (*viz.* 9 feet) against 12 in the middle of the rule (*wrote girt-line*) then against the girt (*viz.* 14) on the girt-line, you have $12\frac{1}{2}$ feet on the slip.

The tenth QUESTION proved.

Set $12\frac{1}{2}$, the length, against 12, the girt-line; then against $10\frac{1}{2}$, you have better than $9\frac{1}{2}$ on the slip itself, *viz.* 9 feet, 82 inches

Of tapering Timber.

Some persons will take but one girt, though a tree be very long and tapering; but this is certainly very wrong, as it may do injustice to either the buyer or feller. The best way is, to measure such a tree, as if it were two or three distinct trees, by taking two or three several lengths and girts.

Some, indeed, take two girts, one at the great, and the other at the small end, and add them together, and take the half of it for a mean girt (as in board measure) but this is a hurt to the buyer, and very erroneous; whereas they should multiply one girt by the other, and extract the square root for a mean girt.

Quest. 11. Suppose a tree 20 inches girt at one end, and 40 at the other, and 9 feet long, I demand the content?

By the customary way the mean girt will be 30 inches, and the content will be 56 feet, 36 inches = $\frac{1}{4}$ of another foot. But, according to the true way the mean girt is but 28.28, and the content but 49.9 feet, *viz.* 49 feet, 141 inches, which is 6 feet 39 inches less than the other.

Tyro. This is a sensible difference indeed, in many loads of timber.

Philo. Very true.

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Note 1. In some countries, 40 feet make a load, and in others, 50 feet make a load.

Note 2. When there are 50 feet to the load, then to cast up the content, at any given price, the rule is, multiply the content or number of feet by the price in shillings, and cut off the three first figures from the right to the left-hand: so will the figures towards the left-hand be pounds sterling, and the other will be decimal parts of a £.

EXAMPLE.

Quest. 12. Suppose I measured 6 trees, and their content be 548 feet, at £. 1 - 10s. per load?

I multiply 548 by 30, and it gives 16440, which I cut off thus, 16|440, and it is £. 16.440, viz. £. 16-8-9½.

N. B. Stone is measured the same, only observe 8 inches make 1 foot of stone.

4. Of BRICK-WORK.

Tyro. How is brick-work measured?

Philo. By the square rod, that is $16\frac{1}{2}$ feet in length, and $16\frac{1}{2}$ in breadth, make $272\frac{1}{4}$ feet, or one square rod; but for common practice, 272 feet only is sufficient.

Tyro. Is there not a certain standard for the thickness of brick-work?

Philo. Yes, all sorts of brick-work is reduced to the standard of $1\frac{1}{2}$ brick thick, of which I shall give you a farther notion by and by.

1. Of Work at $1\frac{1}{2}$ Brick thick the Standard.

Multiply the length by the height in feet, and divide by 272, the quotient gives the square rods, and the remainder the feet or parts of a rod.

Quest. 13. A gentleman built a brick-wall round his garden, which was 998 feet long, 9 feet high, and $1\frac{1}{2}$ brick in thickness: I demand how many rods it contains? *Ans.* 33 rods, 6 feet.

Here I multiply 998, the length, by 9, the height, and it gives 8982 feet, which I divide by 272, (the feet in a rod) and it gives 33 rods, 6 feet. *Ans.*

Tyro. I understand it; this is easy enough: but suppose it was but 1 brick thick, or suppose it were two or three bricks thick, how then?

Philo. Having found the content at $1\frac{1}{2}$ brick thick, as before directed, say thus: As 3 (the $\frac{1}{2}$ bricks in the standard-measure) is to the content in standard-measure, at $1\frac{1}{2}$ brick thick, so is the number of half bricks in the wall to the content, at that thickness.

Quest. 14. Let the wall be 998 feet round, and 9 feet high, as before, what is the content, at $2\frac{1}{2}$ bricks thick?

The content at $1\frac{1}{2}$ thick was found in the last question to be 33 rods, 6 feet. Say therefore, As 3 to 33.6, so is 5 half-bricks, viz. the thickness at $2\frac{1}{2}$ bricks thick, to the content at that thickness, viz. 55 rods, 10 feet.

Tyro. I understand you well; but cannot any thickness be done at one operation?

Philo. Yes; for having multiplied the length by the height, divide by any of the following numbers, that are set against the given thickness, and you have the content in rods at once, and the remainder is feet.

Note. Though there be decimals in the divisors, you may divide by whole numbers for common use.

For 1	} Brick thick divide by	408.3	For $3\frac{1}{2}$	} Bricks thick divide by	116.6
$1\frac{1}{2}$		272.25	4		102.1
2		204.2	$4\frac{1}{2}$		90.7
$2\frac{1}{2}$		163.3	5		81.7
3		136.12			

Quest. 15. There is a wall 15.5 feet long, and 9 feet high; what is the content, at $3\frac{1}{2}$ bricks thick? *Ans.* 1 rod, 2 tenths: for I multiply $15.5 \times 9.5 = 147.25$, which divided by 116.6, the divisor for $3\frac{1}{2}$ bricks, gives 1 rod, 2 tenths: And thus for any thickness: for at $4\frac{1}{2}$ thick, it is 1.5 rod, viz. $1\frac{1}{2}$.

By the Slip or Sliding Rule.

There is a wall 9 feet high, and 76 feet long, and $1\frac{1}{2}$ brick thick; I demand the content? *Ans.* 2 rods, 140 feet, or better than $2\frac{1}{2}$ rods.

Set 272 on the slip to the height 9 above it; then against 76 the length on the slip, is $2\frac{1}{2}$ or better, on the rule.

A RULE for any Thickness.

Set any of the former divisors, answering any thickness on the slip, to the height; then against the length is the answer.

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Thus, the same wall at 3 bricks thick: Set 136 to 9, then against 76 you have 5 rods, the content at 3 bricks thick.

5. Of SURVEYING.

Tyro. How is land surveyed?

Philo. Land-Measure is a part of the mathematics, and to survey it true, and in a masterly manner, you should be provided, 1. With a chain called Gunter's chain. 2. A case of instruments. 3. A parallel ruler. 4. A plain table. 5. A plotting-scale, or protractor. And, to make it more complete, a theodolite.

Tyro. But cannot I measure a common regular field, or little piece of ground, without all these instruments?

Philo. Yes; by a chain only; or for want of that, a cord, a rod-pole, or any such thing; but this must not be depended upon for truth.

Tyro. Give me a description of the chain.

Philo. All land is now generally measured by a chain, containing 4 rods, or poles, in length, (viz. 22 yards) according to a statute made in the 33d of Edward I. anno 1235, which says, that a square acre shall contain 160 rods, viz. 40 rods in length, and 4 rods in breadth, make 160 rods, or 1 acre of ground.

Note. The chain is made of iron, containing 100 links, each in length is 7.92 inches or nearly 8 inches, 100 of which are 792 inches, or 22 yards, (viz. 4 rods) therefore, 1 chain in length, and 10 in breadth, or 10 in length and 1 in breadth, make an acre.

Note 2. For want of a chain you may take a cord 22 yards, or four rods long, or any number of rods long you please, dividing it into halves and quarters, with which you may measure any common field within a trifle of truth, or, at least, for common satisfaction.

Having provided yourself with a chain, or any convenient line, if the field or piece of ground be regular, viz. a square, or the opposite sides alike; then measure the length and the breadth, in rods or parts, and multiply the length by the breadth, and divide the product by 160, the rods in an acre, you have the content.

Quest. 5. There is a field in the form of a long square (called a parallelogram) whose length is 35 rods, and breadth 24 rods, I demand the content in acres? *Ans.* 5 acres 1 rood.

First, I multiply 35, the length, by 24, the breadth, and it gives 840 rods; which I divide by 160, gives 5 acres, and 40 remain, which I multiply by 4 (because 4 roods make 1 acre) and divide again by 160, gives 1 rood.

Quest. 16. There is a square piece of ground set out upon a heath or common, in order to form a camp for 1000 soldiers, each side contains 60 rods: how many acres does it contain? *Ans.* $22\frac{1}{2}$ acres.

For $60 \times 60 = 3600$, which divide by 160 = 22 acres, 80 rods, or $22\frac{1}{2}$ acres.

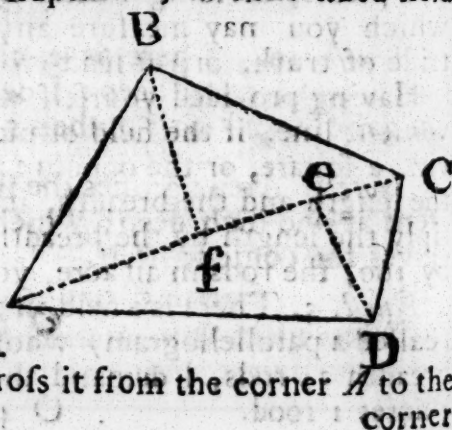
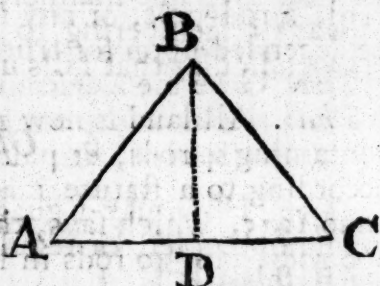
Quest. 17. There is a 3-sided or triangular field, ABC , the side AC is 51.5 rods, and the perpendicular BD is 34 rods; how many acres does it contain? *Ans.* $5\frac{1}{2}$ acres nearly.

Note, You must first of all measure the side from A to C , called the base, which suppose $51\frac{1}{2}$ rods; then measure half way from A to C , and from D measure straight up the point B , which is called the perpendicular, which suppose to be 34 rods: now, I told you before, that the base multiplied by half the perpendicular gives the content; that is 51.5 multiplied by half the perpendicular 17, gives 875.5, which divided by 160, gives 5.47 acres, that is, very near $5\frac{1}{2}$ acres.

To measure any four-sided Field, whose Sides are unequal, called a Trapezium.

Quest. 18. There is a trapezium, or four-sided field or piece of ground, $ABCD$, whose base AC is 64 rods, and the perpendicular Bf is 60, and the other perpendicular De is 40: I demand the content in acres? *Ans.* 20 acres.

First, to measure A his field, go straight across it from the corner A to the corner D



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corner *C*, which here is called the base, and measures 64 rods: then measure right strait from the point *B* to *f*, which is 60 rods, and right strait from *D* to *e*, which is 40 rods. This done, the rule is,

Multiply the whole base *AC* 64 by $\frac{1}{2}$ *Bf* 60 (viz. 30) and it gives 1920 rods, the content of the triangle *ABC*; then again multiply the base *AC* by $\frac{1}{2}$ *De* 40 (viz. 20) and it gives 1280 rods, the content of the triangle *ACD*. Add those two together, viz. 1920, and 1280 rods, gives 3200 rods, which divide by 160, the rods in an acre, gives 20 for the answer.

Tyro. Sir, I thank you; this is enough for my purpose at present.

Philo. If the field has more sides, you may measure it after the same manner, by dividing it into triangles, always remembering to multiply the base by $\frac{1}{2}$ every perpendicular that falls upon it.

6. Of GAUGING.

Tyro. How may I gauge, or tell the content of any common cooler, or regular cask, or cistern, in gallons, or bushels.

Philo. I shall give you some short instructions, by which you may tell the content of several things near enough truth, for your own satisfaction; but to be a practical gauger, you ought to understand several branches of the mathematics.

1. To tell the Content of a Malt Cistern in Gallons and Bushels.

Quest. 19. There is a cistern 6.5 feet long, 4 feet wide, and 3.5 feet deep: I demand its area and content in gallons and malt bushels?

Note 1. Area signifies the superficial content, or content at one inch deep; which multiplied by the depth, gives the content itself.

Note 2. That 282 inches make 1 gallon of ale, water, &c. 231 a gallon of wine, and 2150 inches one bushel, which are your divisors for all regular figures.

R U L E.

Multiply the length, 78 inches, by the breadth, 48, and it gives 3744 inches, which divide by 282, gives 13.276, the area, at one inch deep; or divide by 2150, gives 1.741, the area in bushels. The area multiplied by the depth, 42 inches, gives 557.592 gallons. The area for malt multiplied by 42, gives 73.122, the content in bushels.

Note. If the area be not required, or you do not understand decimals, you may more easily find the content at once, thus: Multiply the length, breadth, and depth, in inches, together, gives 157248, which divide by 282, gives 557½ gallons, 157248 divided by 2150, gives 73 bushels $\frac{1}{10}$ as before.

To find the Area by the Sliding Rule.

Set 282 upon *B* to 48, the breadth, on *A*; then against 78, the length, on *B*, is 13.277, the area in gallons.

For malt, set 2150 on *B* to 48 on *A*; then against 78, the length, on *B*, is 1.741 on *A*, the area in bushels; and thus for any regular figure.

To gauge a Tub or Cooler, in the Form of a Cylinder, viz. whose Top and Bottom Diameters are equal.

R U L E.

Square the diameter, viz. multiply it by itself, and this product by the depth, then divide by 359 for beer gallons, 294 for wine, and 2737 (or rather by 2737.47) for malt.

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Quest. 20. There is a tub 4 feet 2 inches diameter, and 3 feet 4 inches deep: I demand the content in beer, wine, and malt?

I square the diameter 50, which is 2500, and multiply this by the depth, 40 inches, gives 100000; which divide by 359, gives 278 ale gallons; divide by 294, gives 340 wine gallons, and by 2737 only, gives 365 malt bushels.

Note. If the figure be the form of a triangle, or trapezium, (as in *questions 17 and 18*) you must proceed to measure them as there directed, and after having multiplied by the depth, divide 282 for beer, 231 for wine, and 2150 for malt, gives the content.

Quest. 21. There is a tub, whose top diameter is 40 inches, bottom 30 inches, and the depth 60 inches: I demand the content in beer, wine, and malt?

There are several ways to do this. One is this: multiply the diameters together, and extract the square root for a mean diameter, which is here 34.64. This multiplied by itself, and divided by 359, gives the content in gallons, or by 2737, gives the malt bushels.

Or more easily thus, though not so true.

Add the diameters together, and take the $\frac{1}{2}$ for a mean is 35. Now $35 \times 35 = 1225 \times 60 = 73500$, which divided by 359, gives about 205 gallons; and so for malt, viz. 2737, gives 26 bushels $\frac{8}{10}$.

For a Couch of MALT.

1. If it be a regular square only, multiply the length, breadth, and depth together, and divide by 2150, gives the bushels.

2. If it be a triangle, or trapezium, proceed as before directed, and divide still by 2150.

Tyro. But suppose the couch be uneven, how shall I tell where to take the depth?

Philo. Take the depth at four or five places, add them all together, and divide by the number of places you took the depth at, for a mean depth.

Quest. 22. There is a bed or couch of malt, in the form of a long square, whose length is 35 feet, breadth

16 feet, and I find the mean depth to be 8.5 inches, viz. $8\frac{1}{2}$ inches; I demand the content?

Thus, $420 \text{ inches} \times 192 = 80640 \times 8.5 = 685440$; this divided by 2150, gives 318.8 bushels.

Of CASK GAUGING.

There is a great variety in gauging casks; but the following methods will be near enough truth for all common casks; such as barrels, butts, &c. that are pretty much bulged.

First, Having taken the bung and head diameters, the rule is, to the sum and $\frac{1}{2}$ the sum of the squares of the bung and head diameters add $\frac{1}{2}$ the difference of the said squares: this sum multiply by the length, and divide by 1077 for beer, or 882 for wine gallons.

2. *Rule*, which is as true, and much easier.

To the double square of the bung diameter add the square of the head diameter; then multiply this sum by the length of the cask, and divide by 1077 for beer, or 882 for wine.

Quest. 23. There is a cask, whose bung diameter is 28 inches, head diameter 25 inches, length 36: I demand the content in ale gallons?

First, The square of the bung diameter 28 is 784: which doubled is 1568. Then the square of the head, viz. $25 \times 25 = 625$, which added to 1568, is 2193; this $\times 36$, the length, is 78948, which divided by 1077, gives 73 gallons, 2 pints, for beer, and divided by 882, gives 89 gallons and $\frac{1}{2}$, wine or brandy.

Note, If you find the area of the bung and head diameters (by *question 20*) and add twice the area of the bung, viz. 2.184, to the area of the head 1.741, it is 6.109, which multiplied by $\frac{1}{3}$ of the cask's length, viz. 12, gives 73.308 gallons, as before.

These methods holding good for most casks, I shall give no more examples.

Note 2. If one of the head diameters be larger than the other, and the cask is strait in the sides, like some churns, then (by *question 21*) find a mean diameter throughout, and proceed as therein directed.

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7. Of CROSS MULTIPLICATION.

There are two Methods.

1. By MULTIPLICATION only.

1. *Rule.* Feet multiplied by feet produce feet.
2. Inches multiplied by feet, or feet by inches, produce inches.
3. Inches multiplied by inches produce parts.

Note, 12 seconds make 1 part, 12 parts make 1 inch, and 12 inches 1 foot.

2. By MULTIPLICATION and DIVISION.

Rule. Having placed the lesser sum for the multiplier, multiply the very last place of the multiplicand towards the right hand by the first place or name of the multiplier, and carry one for every 12, setting down what is over 12 under the part you multiplied, then take the parts of the multiplier as in practice, carrying as before 1 for every 12.

But an example will render it more easy, if I give it both ways.

	Feet	In.	Parts		Feet	In.
Multiply	4	3	6	by	4	3
by	4	3				
	<hr/>					
	17	2	-			
	1	-	9			
			1	6		
	<hr/>					
Ans	18	2	10	6		
	Feet	In.	P.	Sec.		

First, I begin and multiply the top 4 feet, 3 inches, and 6 parts, by 4 feet, (carrying 1 for every 12) saying, 4 times 6 is 24 parts, that is, 0 and carry 2; then 4 times 3 is 12, and 2 I carried is 14 inches, that is, 2 inches, and carry 1 to the feet; then 4 times 4 is 16 feet and 1 is 17.

Secondly,

Secondly, I multiply now 4 feet, 3 inches, 6 parts, by the lower 3 inches, saying, 3 times 4 is 12 inches (because feet multiplied by inches are inches) then 3 times 3 inches is 9 parts, (for inches by inches produce parts) and lastly, three times 6 parts is 18 parts, viz. 1 part, 6 seconds.

The Second Method.

I first multiply the first or top line as before, and find it as before, 17-2-0; and now I take the parts, as in Practice; saying, 3 inches is $\frac{1}{4}$ of a foot, &c. See the work.

Feet In		P.
4	3	6
4	3	
<hr/>		
In.		
3	$\frac{1}{4}$	17 2 -
		1 - 10 6
<hr/>		
		18 2 10 6 as before.

I say, 3 inches is $\frac{1}{4}$ of a foot, and take the parts from 4 feet, 3 inches, 6 parts, saying, the 4th of 4 is 1 foot; then the 4th of 3 inches, 6 parts, (viz. 42 parts) is 10 times 4 is 40, and 2 parts over; lastly, I say the 4th of 2 parts (viz. 24 seconds) is 6 seconds; which is now done in 2 lines only.

And now, *Tyro*, I must bid you farewell, and I hope you will take care to improve yourself in them, rather than trifling away your time with idle fancies; for it is evident, that the knowledge of arithmetic is necessary in every station of life, since almost all manner of business depends upon it: and not only this, *Tyro*, but it is a great help to protect us against the frowns of fortune, and keep us (by being qualified for some lawful post or employment) from those common temptations and misfortunes, to which those who know the want of it, so often fall into, and pay for so dearly.

Tyro. I return you thanks for this advice, and hope I shall make such use of it as may not frustrate your good designs.

Philo. I make no doubt but you will. And therefore—I once more bid you an hearty farewell.

Tyro. Sir, I am your humble servant.



APPENDIX:

APPENDIX:

CONTAINING

A CONCISE METHOD

OF

BOOK-KEEPING.

RUDIMENTS of BOOK KEEPING.

THE greatest advantage necessary for Youth, must proceed from a right knowledge and practice of keeping orderly accounts.

To answer this end, I have formed an easy scale, whereby any one capable of subtracting one sum of money from another, may state and ballance their own accounts correctly, and have a clear view of all their transactions.

RULES *for knowing the Debtor in all Circumstances.*

Every thing I receive is Debtor to the person from whom received : And every person to whom I give, is Debtor to the thing given.

Again, if I exchange one article for another, the article received is Debtor to the one given.

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W A S T E B O O K.

London, January 1, 1783.

INVENTORY of the READY MONEY, £. s. d.
 GOODS, and DEBTS, belonging to me,
 A. B. taken this day, viz.

	£.	s.	d.
In ready money —	857	10	0
40 bales of broad cloth, at 35 <i>l.</i> per bale —	1400	0	0
12 hhds. of sugar, wt. 175 <i>C.</i> 1 <i>qr.</i> at 32 <i>s.</i> 6 <i>d.</i> per <i>Cwt.</i> }	284	15	7½
150 bags of ginger, wt. 175 <i>C.</i> at 24 <i>s.</i> —	210		
70 anchors of brandy, at 62 <i>s.</i> 6 <i>d.</i> —	218	15	
William James owes me on demand —	80	10	
Abraham Green, per bond, due 4th <i>Feb.</i> next	100		
	3151	10	7½
I owe as follows :			
To John Worthy, on demand 80			
To James Innis, per bond, with interest, from 4th <i>June</i> , 1780 —	500		
	580	0	0

January 2, 1783.

	£.	s.	d.
Bought 17 pipes of sherry of Wm. James, at £. 42 per pipe	714	0	0
Sold 1 hhd. of sugar, wt. 16C. 2 qrs. neat, to John Ingram, at 37s. 6d. per C.	30	18	9
Sold 4 bales of broad cloth, for ready mo- ney, at £. 40 per bale	160	0	0
Sold William Manners 5 hhds. of sugar, weight 70C. 1qr. 18lb. at 38s. per C.	133	15	7½
<hr/> 3. <hr/>			
Bartered with William Davis 10 pipes of sherry, at £. 43 per pipe	430	0	0
For 9 puncheons of rum, quantity 958 gallons, at 6s. 6d.	311	7	0
Bought a silver tea-kettle and lamp, for which I paid	20	0	0
Paid for a set of books for my 'compting- house	5	5	0
Paid my housekeeper her bill of expences	32	14	6½
<hr/> 4. <hr/>			
My warehouse-keeper has informed me of the ullage of ½ the pipe of sherry, bought of Wm. James, which I value at	21	0	0
Sold 4 ankers of brandy, for ready money, at 67s. 6d. per anker	13	10	0
Received by the hands of Thomas Wil- liamson, in full of a legacy left me by my uncle, John Roberts	520	0	0
Paid my book-keeper his quarter's salary, and bill of charges	34	18	9
Sold 2 punch. of rum, quant. 245 galls. at 10s. 6d. per gallon, for ready money	128	12	6
Gave as a benefaction to the Salisbury Infirmary	50	0	0
Sold the remainder of my sherry, being 6 pipes and 1 hhd. at £. 47 per pipe, to William James	305	10	0

N. B. The
Ledger, wh
is put, it d



JOURNAL

N. B. The figures in the left-hand column refer to the folios of the Ledger, where the accounts are transferred to; and when this mark ✓ is put, it denotes that the article is posted in the Ledger.

		London, January 1, 1783.			£.	s.	d.
		Sundry Accounts, £. 3151-10s. 7½d.					
		Dr. to Stock.					
			£.	s.	d.		
✓ 2	Cash in ready money		857	10			
✓ 1	Broad cloth for 40	}	1400	0	0		
	bales, at £. 35 per						
	bale						
✓ 3	Sugar for 175 C. 1 qr.	}	284	15	7½		
	at 32s. 6d. per C.						
✓ 3	Ginger, for 175 C. at	}	210	0	0		
	24s. per C. —						
✓ 2	Brandy, for 70 ankers,	}	218	15	0		
	at 62s. 6d. —						
✓ 2	Wm. James, on demand		80	10	0		
✓ 3	Abraham Green, per	}	100	0	0		
	Bond, due 4th Feb.						
	next —						
			3151	10	7½		
1	Stock, £. 580 to sundry Accounts.						
✓ 3	To John Worthy, on	}	80	0	0		
	demand —						
✓ 3	To James Innis, per	}	500	0	0		
	bond, with interest,						
	from June 4, 1780						
			580	0	0		

		January 2, 1783.		£.	s.	d.
✓	$\frac{4}{2}$	Sherry, £.714 to William James, For 17 pipes, at £.42	—	714	0	0
✓	$\frac{4}{3}$	John Ingram, £.30 18s. 6½d. to sugar For 1 hhd. wt. 16C. 2 qrs. at 37s. 6d.	—	30	18	9
✓	$\frac{2}{1}$	Cash £.160 to broad cloth For 4 bales, at £.40	—	160	0	0
✓	$\frac{4}{3}$	Wm. Manners, £.133 15s. 7¼d. to sugar For 5 hhds. wt. 70C. 1 qr. 18lb. at 38s. per C.	—	133	15	7¼
		3.				
✓	$\frac{4}{4}$	William Davis, £.430 to sherry* For 10 pipes, delivered to him in barter, at £.43	—	430	0	0
✓	$\frac{4}{4}$	Rum, £.311-7s. to William Davis* For 9 puncheons, quant. 958 galls. at 6s. 6d. received in barter	—	311	7	0
✓	$\frac{5}{2}$	Plate and jewels, £.20 to cash Paid for a tea-kettle and lamp	—	20	0	0
✓	$\frac{5}{2}$	Charges of merchandize, £.5 - 5s. to cash Paid for a set of books for the 'compting-house	—	5	5	0
✓	$\frac{5}{2}$	House expences, £.32 - 14s. 6½d. to cash Paid my house-keeper's bill of ex- pences	—	32	14	6½
		4.				
✓	$\frac{1}{4}$	Profit and Loss, £.21 to sherry For the ullage of ½ a pipe	—	21	0	0

* These are two Journal Posts in the case of Barter.

JOURNAL.

3

January 4, 1783.

			£.	s.	d.
✓	$\frac{2}{2}$	Cash £.13 - 10s. to brandy			
		For 4 ankers, at 67s. 6d. per anker	13	10	0
✓	$\frac{2}{1}$	Cash £.520 to Profit and Loss			
		Received a legacy left me by my uncle Roberts	520	0	0
✓	$\frac{5}{2}$	Charges of merchand. £.34 - 18s. 9d. to cash			
		Paid my book-keeper a quarter's salary, and bill of charges	34	18	9
✓	$\frac{2}{4}$	Cash £.128 - 12s. 6d. to rum			
		For 2 punch. 245 galls. at 10s. 6d.	128	12	6
✓	$\frac{1}{2}$	Profit and Loss, £.50 to cash			
		Gave to the <i>Salisbury</i> Infirmary	50	0	0
✓	$\frac{2}{4}$	William James, £.305 - 10s. to sherry			
		For 6 pipes and 1 hhd. at £.47 per pipe	305	10	0

L E D G E R.

A

G

Ginger
Green, Abraham

3
3

B

Broad Cloth
Brandy
Balance

1
2
5

H

House Expences

5

C

Cash
Charges of Merchand.

2
5

I

James, William
Innis, James
Ingram, John

2
3
4

D

Davis, William

4

K

E

L

F

M

Manners, William

4

Profit a
Plate a

Rum

Stock
Sugar
Sherry

	N		T	
3 3	O		U	
5	P		W	
	Profit and Loss Plate and Jewels	1 5	Worthy, John	3
2 3 4	Q		X	
	R		Y	
	Rum	4		
4	S	1 3 4	Z	
	Stock Sugar Sherry			

LEDGER.

		Folio in Journal		Folio in Ledger	£.	s.	d.
Dr.—Stock.							
1783 Jan. 1.	1	To sundry accounts, as per Journal			580	00	
		To balance for the neat of my estate		5	3083	12	1
					3663	12	1
Dr.—Profit and Loss.							
1783 Jan. 4.	2	To sherry for 1 hhd. ullaged		4	21	00	
	3	To cash gave the Salisbury Infirmary		2	50	00	
		To house expences		5	32	14	6½
		To charges of merchandize		5	40	39	
		To stock gained by trade		1	512	1	5½
					655	19	9
Dr.—Broad Cloth.							
1783 Jan. 1.	1	To stock, at £. 35 per bale, for 40 bales		1	1400	00	
		To profit and loss gained		1	20	00	
					1420	00	

1783
Jan. 11783
Jan. 41783
Jan. 2.

LEDGER.

1

Per Contra—Cr.

1783
Jan. 1.

Folio in Journal

1 By fundry accounts, as per
Journal
By profit and loss, gained by
trade

Folio in Ledger

£.	s.	d.
315	10	7½
1	512	15½
366	3	1

Per Contra—Cr.

1783
Jan. 4.

3

By cash for a legacy left by uncle
Roberts
By broad cloth gained
By brandy gained
By sugar gained
By sherry gained
By rum gained

2
1
2
3
4
4

520	00
20	00
1	00
23	99
42	00
49	00
655	199

Per Contra—Cr.

1783
Jan. 2.

2

By cash at £.40 per } 4 bales
bale, for
By bal. remaining at } 36
£.35 per bale, for

2
5

160	00
1260	00
1420	00

LEDGER.

				Fol. in Jour.				Fol. in Ledg.	£.	s.	d.	
Dr.—Brandy.												
1783	Jan. 1.	1		1	To stock, at 62s. 6d. per anker,			1	218	15	0	
					for 70 ankers	—		1	1	0	0	
					To profit and loss gained							
										219	15	0
Dr.—Cash.												
1783	Jan. 1.	1		1	To stock in ready money			1	857	10	0	
	2.	2		1	To broad cloth for 4 bales, at			1	160	0	0	
					£.40 per bale	—						
	3.	3		2	To brandy for 4 ankers, at 67s.			2	13	10	0	
					6d. per anker	—						
				1	To profit and loss, received a			1	520	0	0	
					legacy left me by uncle Ro-							
					berts	—		4	128	12	5	
					To rum, for 245 gall. at 10s. 6d.							
										1679	12	6
Dr.—William James.												
1783	Jan. 1.	1		1	To stock, per note on demand			1	80	10	0	
	4	3		4	To sherry, for six pipes, 1 hhd.			4	305	10	0	
					at £. 47 per pipe	—		5	328	0	0	
					To balance due	—						
										714	0	0

1783
Jan. 41783
Jan. 31783
Jan. 1

LEDGER.

2

		Fol. in Jour.		Fol. in Ledg.	£.	s.	d.
Per Contra—Cr.							
1783	Jan. 4.	3	By cash at 67s. 6d. per anker, for	2	13	10	0
			By balance remaining at 62s. 6d. per anker	5	206	5	0
					219	15	0

Per Contra—Cr.							
1783	Jan. 3.	2	By plate and jewels, for one tea-kettle and lamp	5	20	0	0
			By house expences, paid house-keeper's bill	5	32	14	6½
			By charges of merehandize, paid for 1 set of books	5	5	5	0
			By ditto, paid book-keeper's salary, and bill of charges	5	34	18	9
4.	3		By profit and loss, gave the Salisbury Infirmary	1	50	0	0
			By balance remaining in my hands	5	1536	14	2½
					1679	12	6

Per Contra—Cr.							
1783	Jan. 2.	2	By sherry, for 17 pipes, at £.42	4	714	0	0
					714	0	0

LEDGER.

		Fol. in Jour.		Fol. in Ledg.	£.	s.	d.
Dr.—Sugar.							
1783	Jan. 1.	1	To stock, at 32s. 6d. per C. for 175 C. 1 ^{qr} .	1	284	15	7½
		1	To profit and loss gained	1	23	9	9
					308	5	4½
Dr.—Ginger.							
1783	Jan. 1.	1	To stock, at 24s. per C. for 175 C.	1	210	00	
Dr.—Abraham Green.							
1783	Jan. 1.	1	To stock, per bond, due the 4th of Feb. next	1	100	00	
Dr.—John Worthy.							
1783	Jan.		To balance due	5	80	00	
Dr.—James Innis.							
1783	Jan.		To balance due	5	500	00	

LEDGER.

3

Per Contra—Cr.

1783		C. qr. lb.				
Fol. in Jour.				Fol. in Ledge.	£.	s. d.
Jan. 2	2	By John Ingram, 37s.				
		6d. per C. for	16 2 0	4	30	18 9
		By Wm. Manners,				
		38s. per C. for	70 1 18	4	133	15 7 ¹ / ₄
		By balance rem. at				
		32s. 6d. for	88 1 10	5	143	11 0 ¹ / ₄
			175 1 0		308	5 4 ¹ / ₂

Per Contra—Cr.

1783						
Fol. in Jour.				Fol. in Ledge.	£.	s. d.
Jan.		By balance remaining at 24s.				
		for 175 C.		5	210	0 0

Per Contra—Cr.

1783						
Fol. in Jour.				Fol. in Ledge.	£.	s. d.
Jan.		By balance due		5	100	0 0

Per Contra—Cr.

1783						
Fol. in Jour.				Fol. in Ledge.	£.	s. d.
Jan.		By stock, on demand		1	80	0 0

Per Contra—Cr.

1783						
Fol. in Jour.				Fol. in Ledge.	£.	s. d.
Jan.		By stock, per bond, dated 4th				
		June, 1780		1	500	0 0

LEDGER.

	Fol. in Jour.		Fol. in Ledg.	£.	s.	d.
Dr.—Sherry.						
1783 Jan. 2.	2	To Wm. James, at £.42 per pipe, for 17 pipes —	2	714	00	
		To profit and loss gained		42	100	
				756	100	
Dr.—John Ingram.						
1783 Jan. 2.	2	To sugar, for 1 hhd. wt. 16 C. 2 qrs. at 37s. 6d. —	3	30	189	
Dr.—William Manners.						
1783 Jan. 2.	2	To sugar, for 5 hhds. wt. 70 C. 1 qr. 18 lb. at 38s. —	3	133	15	7 $\frac{3}{4}$
Dr.—William Davis.						
1783 Jan. 3	2	To sherry, for 10 pipes, at £.43 dd. in barter —	4	430	00	
Dr.—Rum.						
1783 Jan. 3	2	To W. Davis, received in barter, at 6s. 6d. 958 gall. —	4	311	70	
		To profit and loss gained	1	49	00	
				360	70	

LEDGER.

4

d.		Fol. in Jour.			Fol. in Ledg.	£.	s.	d.
			Per Contra—Cr.					
1783				P. H.				
Jan. 3.	2		By W. Davis, dd. in bar-ter, at £.43 for	10 0 4	430	00		
	4.	2	By profit and loss, for an ullage of	0 1 1	21	00		
	3		By W. James, at £.47 for	6 1 2	305	100		
				17 0	756	100		
			Per Contra—Cr.					
1783								
Jan.			By balance due	5	30	189		
			Per Contra—Cr.					
1783								
Jan.			By balance due	5	133	157 $\frac{1}{4}$		
			Per Contra—Cr.					
1783								
Jan. 3	3		By rum, for 958 gall. at 6s. 6d.	4	311	70		
			By balance due	5	118	130		
					430	00		
			Per Contra—Cr.					
1783				Gall.				
Jan. 4.	3		By cash, at 10s. 6d. for	245 2	128	126		
			By bal. rem. at 6s. 6d. for	713 5	231	146		
				958	360	70		

LEDGER.

	Fol. in Jo.		Fol. in L.	£.	s.	d.
1783 Jan. 3.	2	Dr.—Plate and Jewels.				
		To cash, for tea-kettle and lamp	2	20	0	0
1783 Jan. 3.	2	Dr.—House Expences.				
		To cash, paid my housekeeper's bill	2	32	14	6 $\frac{1}{2}$
1783 Jan. 3.	2	Dr.—Charg. of Merchand.				
		To cash, paid for a set of books	2	5	5	0
4.	3	To cash, paid book-keeper's salary and bill	2	34	18	9
				40	3	9
		Dr.—Balance.				
		To cash remaining	2	153	6	14 $\frac{1}{2}$
		To broad cloth rem. 36 bales, at £.35	1	1260	0	0
		To brandy rem. 66 ankers, at 62s. 6d.	2	206	5	0
		To sugar rem. 88 C. 1 qr. 10 lb. at 32s. 6d.	3	143	11	0 $\frac{1}{4}$
		To ginger rem. 175 C. 1 qr. at 24s.	3	210	0	0
		To Abraham Green, per bond, due 4th of Feb. next	3	100	0	0
		To Wm. Manners, on demand	4	133	15	7 $\frac{1}{4}$
		To Wm. Davis, on demand	4	118	13	0
		To rum rem. 713 gall. at 6s. 6d.	4	231	14	6
		To John Ingram, on demand	4	30	18	9
		To plate and jewels	5	20	0	0
				3991	12	1

1783
Jan.1783
Jan.1783
Jan.

LEDGER.

5

d.	Fol. in Jo.		Fol. in L.	£.	s.	d.
0	1783 Jan.	Per Contra—Cr. By balance remaining —	5	20	00	
6 $\frac{1}{2}$	1783 Jan.	Per Contra—Cr. By profit and loss —	1	32	14	6 $\frac{1}{2}$
0	1783 Jan.	Per Contra—Cr. By profit and loss —	1	40	39	
2 $\frac{1}{2}$		Per Contra—Cr. By Wm. James, on demand	2	328	00	
0		By John Worthy, ditto	3	80	00	
0		By James Innis, <i>per</i> bond, dated 4th June, 1780 —	3	500	00	
0 $\frac{1}{4}$		By stock, the neat of my estate	1	3083	12	1
0				3991	12	1
7 $\frac{1}{4}$						
0						
6						
9						
0						
1						
1						

Necessary Observations.

The Debtor side of the Account of Balance (see fol. 5) contains what goods remain in your hands unfold, what debts are due, what cash you have in your possession, and whatever else of worth belong to you, from which is always formed the Inventory for a new Waste Book. It is also an account of your ready money, goods, debts, &c. and the Credit side an account of the several sums you owe.

After the Account of Stock (which is the last closed but the above) is made Debtor to Balance for the neat of your estate, and the Account of Balance credited by Stock for the same sum, you have then nothing to do but add your Credit side of Balance, and if it amounts to the same sum as the Debtor side, you are certain of its being true.

The last article on the Credit side of Balance is always your present worth.

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The last article on the Credit side of Balance is always your present worth.

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